EFFECT OF UNSUPPORTED TIES AT TRANSITION ZONES

T.D. Stark and S.T. Wilk
University of Illinois at Urbana-Champaign
Urbana, IL

H.B. Thompson II
Federal Rail Administration
Washington D.C

and

T.R. Sussmann Jr.
University of Hartford and
John A. Volpe Transportation Center
Cambridge, MA

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ABSTRACT
Recent investigations of bridge transition zones experiencing reoccurring track geometry issues have exhibited unsupported ties along the approach near the bridge deck. These unsupported ties are identified as problematic as they can increase the loading of the ballast by mechanisms of impact loads and load redistribution. The development of unsupported ties occurs from the approach track lying on deformable layers while the bridge track lies on a rigid, non-deformable structure. Once the approach track settles, the rail in the approach remains supported by the bridge deck and produces a gap between the tie and underlying ballast. Using a three-dimensional numerical model of a bridge approach, the effect of these tie-ballast gaps is investigated. Results show significant amplifications of tie loads in the approach due to the impact of the tie hitting the ballast and redistribution of load.

INTRODUCTION

The expensive and reoccurring problem of differential movement, i.e. transient and permanent vertical displacements, at railway and highway bridge transition zones is a problem across the world (Stark et al., 1995; Long et al., 1999; Nicks, 2009; Coelho, 2011; Mishra et al., 2012). Typically the approach substructure settles at a greater rate than the surrounding track resulting in rail profile deviation, e.g. dips in rail elevation, within the transition zone. In railroads, this represents a safety concern and the problem is especially important because of the expansion and upgrade of high-speed passenger rail, which is more sensitive to sudden changes in rail elevation. The source of this differential movement is often attributed to the stiffness difference between the transition zone track and bridge which increases the dynamic load within the transition zone (Nicks, 2009)

Many recent investigations into the cause of these differential movements have shown a relationship between tie-ballast gaps, e.g. unsupported ties, and increased permanent vertical displacements in the transition region (Namura and Suzuki, 2007; Coelho et al., 2011; Varandas et al., 2011; Wilk et al., 2015). The existence of unsupported ties can further increase the applied loads on the ballast from: (1) the momentum of the moving tie impacting the ballast (Stark et al., 2015) and (2) redistribution of load from poorly supported ties to better supported ties. Ideally, the increase in tie-ballast load from poorly supported ties could be directly measured with pressure plates or pads but this option is problematic because of cost and difficulty of setting up an instrumentation system that covers the entire tie. A second option is indirectly measuring the tie-ballast load by instrumenting well- and poorly supported ties with accelerometers. This has been performed by Stark et al. (2015) and larger peak accelerations were observed at poorly supported ties than their well supported counterparts (30g v. 5g).

A third option is using numerical modeling techniques to investigate how tie-ballast gaps can redistribute and increase tie-ballast loads within both open track and bridge approach track locations. This paper presents the numerical modeling of a passing bogie representing a high-speed passenger train and its effect on track behavior. Various tie-ballast gaps combinations are modeled to gain insight into how tie-ballast gaps can increase the load onto the ballast within transition zones and explain changes in track behavior with time.

TIE-BALLAST GAPS

Tie-ballast gaps can develop throughout the track but are expected to be more prevalent at locations displaying abrupt changes in transient displacement and stiffness (Dahlberg, 2010). Bridge transition zones represent an extreme example where the approach materials displace while the bridge deck remains essentially rigid, resulting in permanent settlement of the approach track substructure. In open track, tie-ballast gaps can develop from uneven track compaction (Dahlberg, 2010) or local regions of ballast fouling but the differential transient displacement is less because all areas of open track are not rigid like a bridge deck.

The formation of tie-ballast gaps involves multiple steps. Initially, assuming the track is recently tamped, the first few wheel loads of a train will transiently and permanently compact the ballast by particle rearrangement and crushing. Due to uneven dynamic loads and ballast settlement, this results in varying substructure elevation along the track, especially at transition zones. Once the substructure settles more at one region of the track than a surrounding region, the high rail bending stiffness causes the rail to pull the ties located over the more settled regions upward, producing a gap underneath the tie. This leaves the tie "hanging" or "cantilevering" from the better supported ties or less settled regions of track. This means tie-ballast gaps develop in the regions of greatest substructure settlement and is often accompanied by a "dip" in rail elevation because of the hanging rail (Paixão et al., 2013; Stark et al., 2015). This is illustrated in Figure 1 which displays the substructure settlements, tie-ballast gaps, and rail profile deviation.

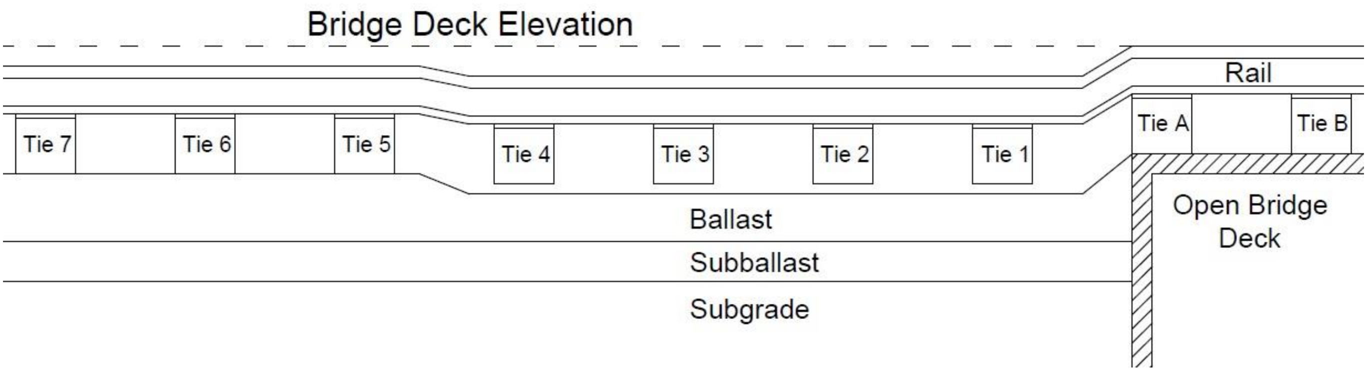

Figure 1: Example of railway bridge transition zone with tie-ballast gaps, substructure settlement, and rail profile deviation.

The schematic in Figure 1 is an example of tie-ballast gaps but variations in tie-ballast gap height can differ within the transition zone region (Lundqvist and Dahlberg, 2005; Varandas et al., 2011). For example, the authors have observed situations where all ties in a transition zone are poorly supported and other situations where a single well supported tie is located a couple meters (6 ft.) in the transition zone surrounded by poorly supported ties. The authors have also observed tie-ballast gap heights increasing with time (Stark et al., 2015) and other situations where the tie-ballast gap height decreases with time. The goal of the numerical simulations is to explain this behavior by investigating how the wheel load is distributed and amplified in the transition zone and how this can lead to uneven substructure settlement.

NUMERICAL MODEL

The finite element software LS-DYNA was selected to numerically model the open track and bridge transition at the Upland Street Bridge near Chester, Pennsylvania and investigate track load redistribution. LS-DYNA is a three-dimensional finite element method (FEM) program distributed by Livermore Software Technology Corporation (LSTC) that specializes in non-linear transient dynamic finite element analyses. LS-DYNA is capable of modeling the entire track behavior along with the inclusion of train cars, wheel systems, rail, tie, and substructure layers.

This paper investigates both open track and bridge approach situations and the finite element meshes are displayed in Figures 1 and 2. The open track mesh (Figure 2) consists of a cart representing the secondary suspension system of a train, 136-RE rail, 32 concrete ties, and a five layer substructure that represents the physical substructure of an instrumented high-speed passenger Amtrak site near Chester, PA. Table 1 presents the sublayer thicknesses and modulus values for each layer estimated using an inverse analysis to match field and numerical data. Of the 32 ties incorporated in the model, only Ties 16 through 24 are of interest in the analysis and the main emphasis on Tie 19.

Table 1: Substructure layer thickness and moduli used in both the open track and bridge approach numerical models.

		Layer 1	Layer 2	Layer 3	Layer 4	Layer 5
Thickness	**[mm]**	**305**	**127**	**508**	**721**	**849**
Modulus	**[MPa]**	**207**	**67**	**33**	**32**	**59**

The cart is modeled to represent the secondary suspension system of a high-speed passenger train. It consists of four wheels with the axles spaced 2.8 m (9.33 ft.) apart to replicate the first bogie of a single Amtrak Acela power car. The cart mass is contained in the cart center with a density such that each wheel applies a static wheel load of 100 kN. The axles and cart mass are connected with four sets of vertical and horizontal springs and vertical dampers. The values of the vertical and horizontal springs are 7.3e5 N/m and 2.2e9 N/m and damper values are set to 7.3e6 N*s/m. The velocity of the cart is inputted as 177 km/hr (110 mph) to replicate the operating speed of Amtrak's high speed trains along the Northeast Corridor (NEC).

The rail geometry is modeled after 136-RE rail (Figure 4) with the density, Young's Modulus, and Poisson's Ratio representing the steel in a 136-RE rail, which are 7.85 g/cm^3, 200 GPa, and 0.28, respectively. The rail-wheel contact is displayed in Figure 4. The concrete ties have a spacing of 0.6 m (2-ft), a width of 0.23 m (0.75-ft), and density, Young's Modulus, and Poisson's Ratio matching the values of concrete, which are 2.97 g/cm^3, 21 GPa, and 0.15 respectively. All model boundaries have pinned and non-reflective boundary conditions. The non-reflective boundary conditions absorb pressure and shear waves, preventing the pressure and waves from reflecting back into the model. The distances to the boundaries are sufficient to prevent boundary effects from influencing the model.

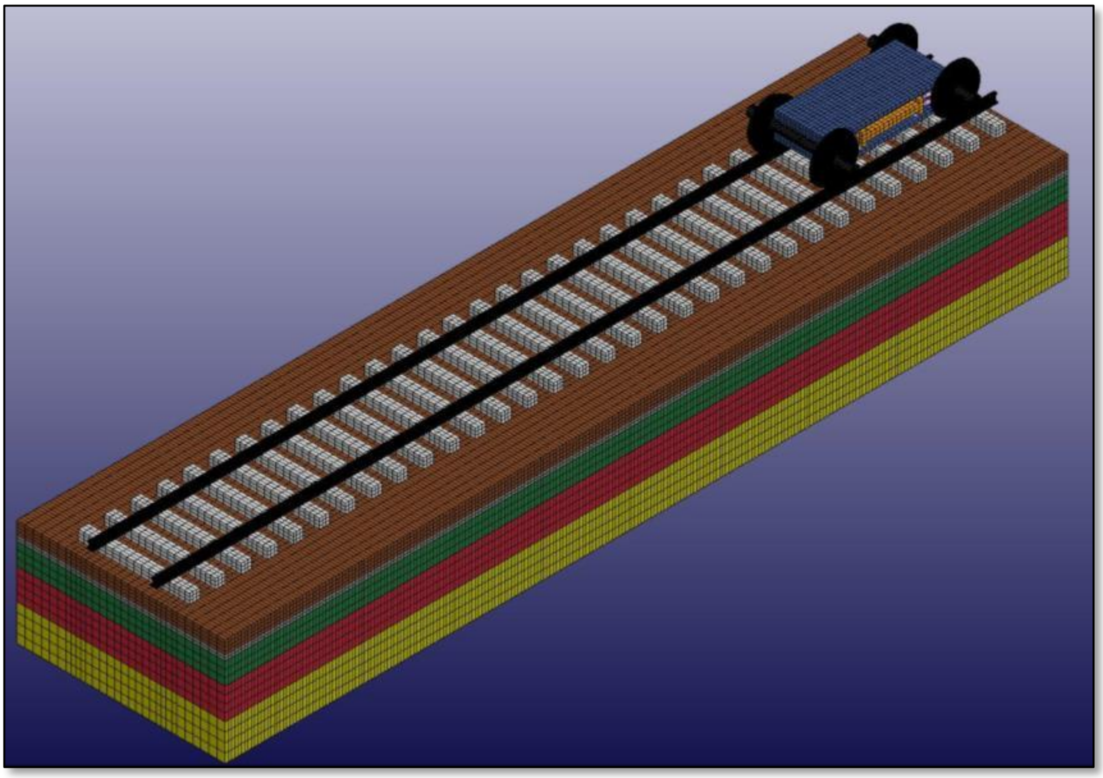

Figure 2: LS-DYNA Finite Element Mesh showing Upland Street Bridge Open Track Site with a Rolling Cart.

The second scenario involves a bridge approach transition zone instead of open track. The bridge is modeled after the Upland Street Bridge in Chester, PA and includes a masonry wall, an open deck bridge with timber ties on the bridge, and W-beams underneath the bridge. The stiffness of the bridge is greater than the approach track which is expected to produce impact loads when the front wheels of the cart pass onto the bridge abutment.

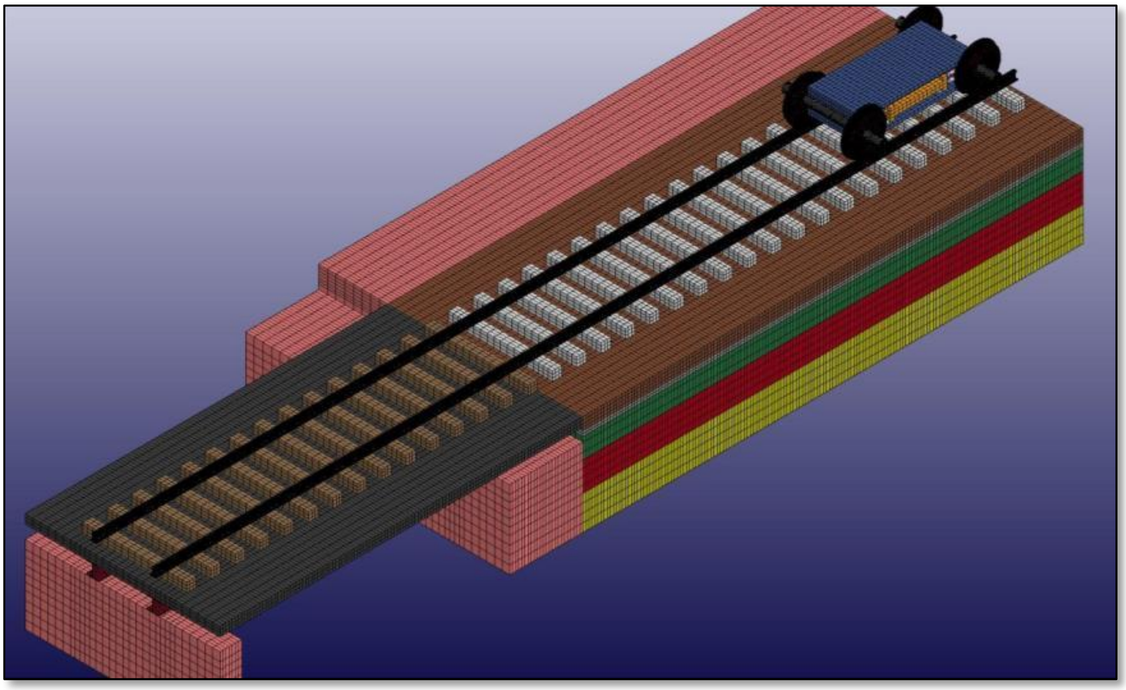

Figure 3: LS-DYNA Finite Element Mesh showing Upland Street Bridge Approach Site with a Rolling Cart.

To simplify the track modeling, the only assumed track defects are tie-ballast gaps, where the bottom of the tie and top of the ballast are simulated as separate entities with contact surfaces (see Figure 5). The tie-ballast gap height is also assumed to be constant under a single tie. While this paper focuses on the tie-ballast gap, the mechanism of load redistribution can occur from any gap or defect within the track system, i.e. rail-fastener gap, fastener-tie gap, or from sudden changes in substructure modulus (Dahlberg, 2010). Also, unsupported or poorly supported ties will likely have differential gaps and stiffness along the length of a single tie, which can influence load distribution under a single tie.

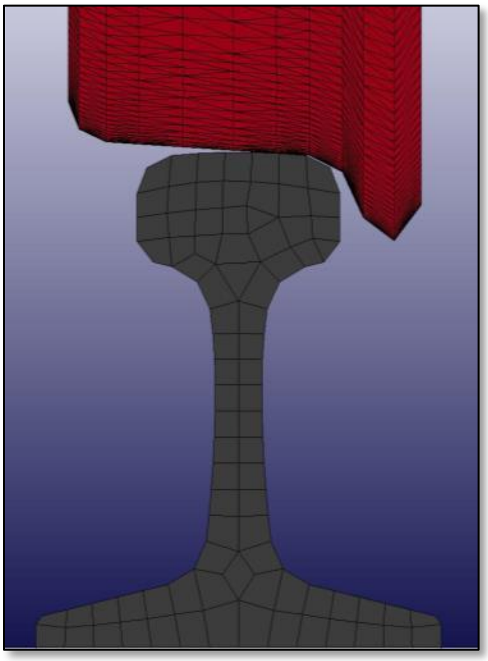

Figure 4: LS-DYNA Element Mesh of Rail and Wheel

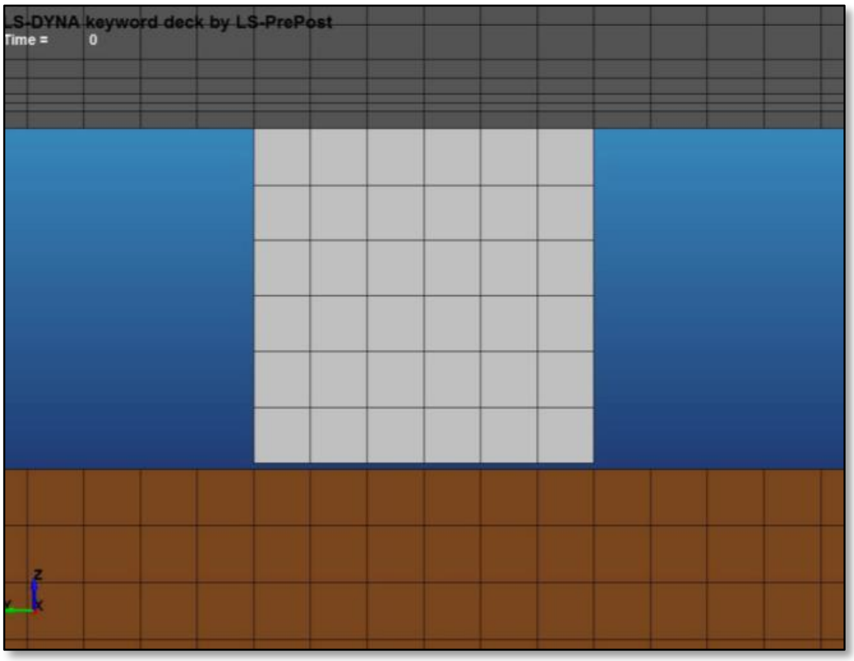

Figure 5: LS-DYNA Element Mesh of Tie-Ballast Gap

The two model outputs from the finite element mesh are wheel-rail contact force and tie-ballast contact forces. The wheel-rail and tie-ballast contact forces are calculated using master-slave penalty methods. This method checks for penetration of slave surfaces, i.e. top of rail surface, through the master surface, i.e. bottom of wheel surface, and applies a proportional force to resist the penetration. These forces are defined as wheel-rail and tie-ballast contact forces.

LOAD DISTRIBUTION – OPEN TRACK

The open track scenario is presented first to isolate and illustrate the mechanism of load redistribution. When a tie-ballast gap develops in open track, the load redistributes from poorly supported ties to surrounding well-supported ties. By increasing the tie load on surrounding ties, those ties may become damaged or develop tie-ballast gaps, spreading the problem to a surrounding track, called "progressive loss of tie support" or "progressive tie failure". The existence of tie-ballast gaps in open track is prevalent in areas of fouling where the fouled section of track will experience greater substructure settlement than the surrounding non-fouled areas.

Load distribution of the wheel load to the underlying ties occurs because the rail, tie, and track substructure are deformable materials. If the tie and track substructure were perfectly rigid, 100% of the wheel load would be transferred to the underlying tie. On the other hand, if no substructure support existed underneath a tie, 100% of the wheel load would be transferred to surrounding ties, meaning the underlying tie receives 0% of the load. Because typically the tie and substructure are not perfectly rigid but do offer support, only about 40% of the maximum applied load is transferred from the rail to the underlying tie in ideal conditions (Chang, 1980; AREMA, 2012). This percentage can vary because the percent wheel load transferred to the underlying tie is affected by track modulus, rail stiffness, and tie spacing (Hay, 1982). Greater track modulus and tie spacing increases the percent wheel load while increases in rail stiffness lowers the percentage.

When a tie-ballast gap develops, the additional displacement required to establish contact with the ballast causes the wheel load to redistribute to surrounding better supported ties, decreasing the percent wheel load from 40%. If adjacent ties develop tie-ballast gaps, the wheel load will not be supported by the adjacent ties and the percent wheel load increases above 40%. To illustrate the effect of load redistribution in open track, a parametric analysis is performed where the tie-ballast gaps of three ties are varied between gap heights of 0.0 to 1.0 mm. The three ties susceptible to tie-ballast gaps are labeled Tie 18, Tie 19, and Tie 20 (see Figure 7) with the gap heights of Tie 18 equaling Tie 20 but not necessarily equaling the tie-ballast gap height at Tie 19. For example, Figure 6 presents a situation where the gap underneath Tie 19 is greater than the gaps underneath Tie 18 and 20.

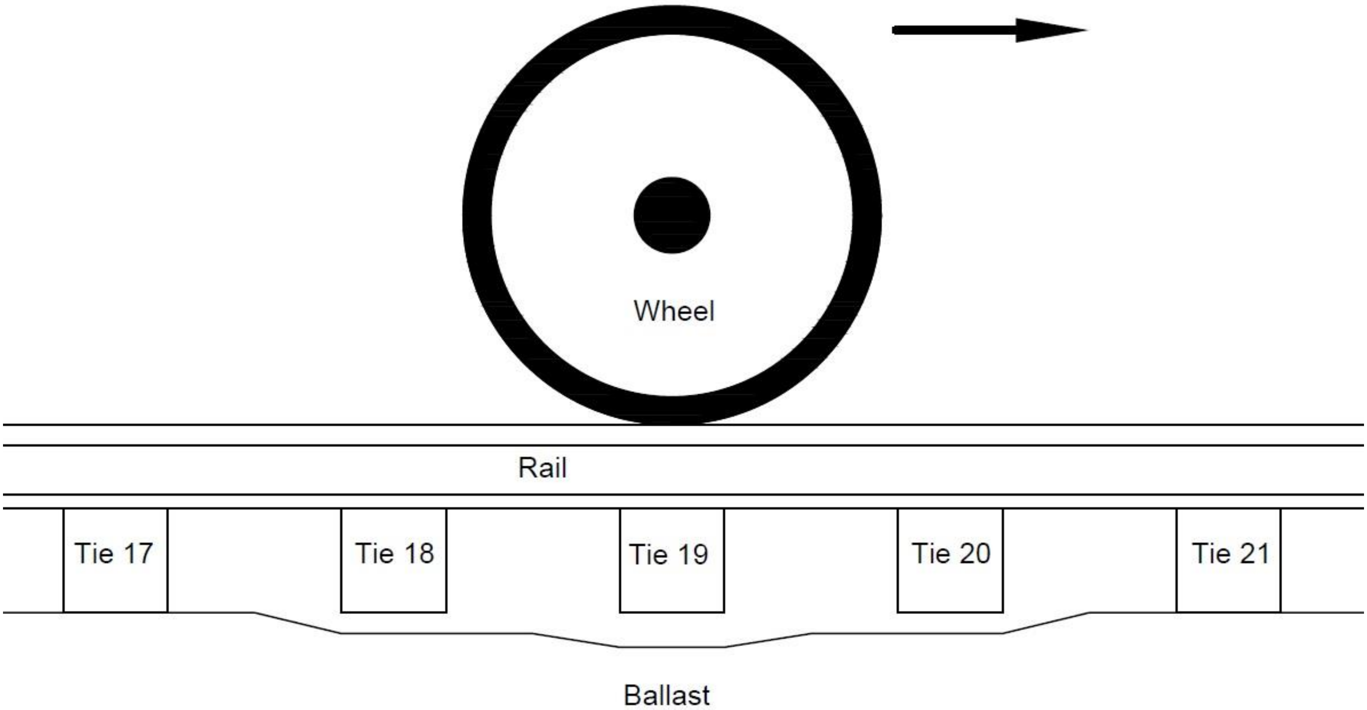

Figure 6: Wheel-Rail-Tie-Ballast Model showing Unequal Tie-Ballast Gaps at Tie 18, 19, and 20.

Figure 7 shows the variation in maximum percent wheel load experienced by Tie 19 (0 – 75%) from a wide range of tie-ballast gap situations. For example, if a tie-ballast gap only exists under Tie 19 (blue diamonds in Figure 7), the maximum percent wheel load of Tie 19 decreases from an ideal 40% when all ties are in intimate contact to zero as load is being redistributed away from Tie 19 to surrounding ties (Ties 18 and 20). If tie-ballast gaps exist only under Ties 18 and 20 (Tie 19 gap = 0.0 mm on x-axis of Figure 7), the maximum percent wheel load experienced by Tie 19 increases from 40% to about 73%, an 83% increase. For gaps under Ties 18, 19, and 20, the load redistribution is sensitive to slight changes in tie-ballast gap height.

These results illustrate how sensitive the distribution of load is to the underlying track conditions and tie-ballast gaps at or even below 1.0 mm. This gap height of 1.0 mm is in agreement with field measurements which relate track locations experiencing greater permanent vertical displacements to gap heights at or above 1.0 mm (Stark et al., 2015) and also laboratory ballast box tests performed by Selig and Waters (1994) which shows increased ballast settlements from tie-ballast gaps at or above 1.0 mm. Because tie-ballast gaps exist at even well performing sites and can vary from tie-to-tie, designing the load at 30 –

50% the wheel load may not be representative of the poorly supported conditions that railroad track actually experience.

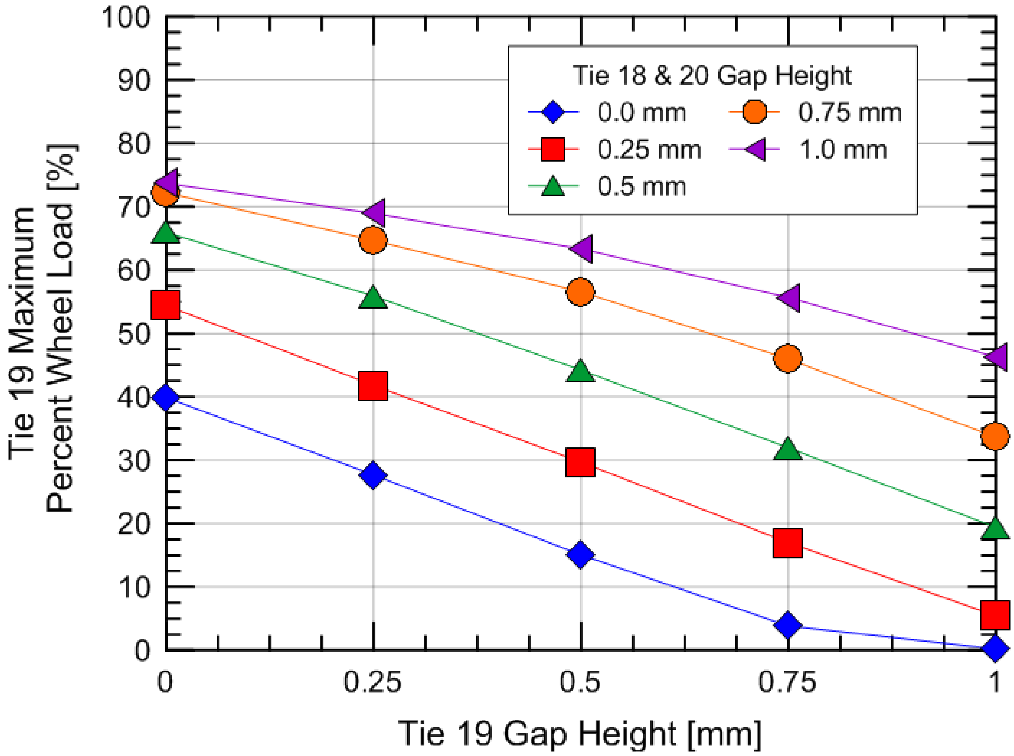

Figure 7: Maximum percent wheel load (maximum tie-ballast contact force / wheel load) for Tie 19 with varying Tie 18, 19, and 20 gap heights

LOAD DISTRIBUTION – BRIDGE APPROACH

The simulation of a bridge approach is more complicated than open track because the transition zone loading is affected not only by: (1) the tie-ballast gaps within the approach but also (2) impact loads resulting from the abrupt change in wheel elevation/track stiffness as the front wheel passes the bridge abutment (Nicks, 2009). The ties of interest are the seven closest ties to the bridge abutment within the transition zone. These are labeled as Ties T1 through T7 and shown in Figure 8. The ties on the bridge are labeled as Tie B1, etc. to prevent confusion.

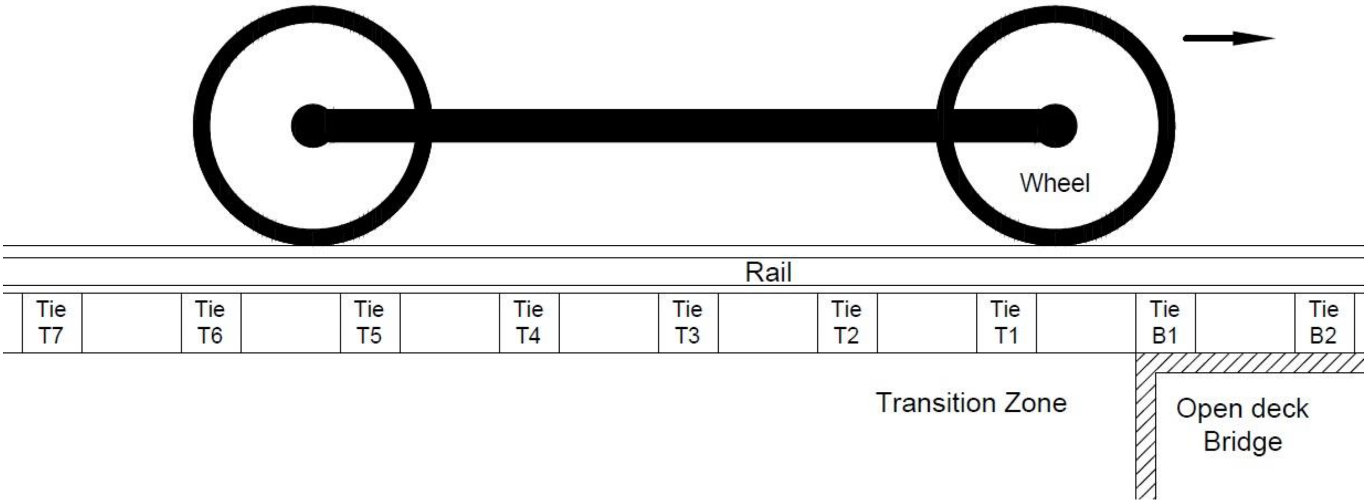

Figure 8: Tie Layout for the Bridge Approach Analyses.

This paper presents three different bridge approach analyses. The first analysis assumes intimate contact, e.g. no tie-ballast gaps, to illustrate the increase in tie-ballast loads from the cart passing the bridge abutment. The two subsequent analyses investigate the sensitivity and redistribution of load from two different tie-ballast gap scenarios. The first scenario assumes that ties closer to the bridge abutment will develop larger tie-ballast gaps. This scenario was observed by Varandas et al. (2011) and Thompson et al., (2015). The second scenario assumes that uneven ballast compaction leaves a single tie in intimate contact with the ballast while the remaining ties develop tie-ballast gaps. Similar scenarios have also been observed by the authors at various track transition zone locations.

Bridge Approach – No Gaps

The first simulation of the cart passing over the bridge approach assumes no tie-ballast gaps are present. This isolates the effect of the impact load (Nicks, 2009) and is imperative for understand more complicated situations when tie-ballast gaps are included. Figure 9 displays the four wheel-rail contact forces with (a) time and (b) distance. Figure 9(a) shows that the front wheels of the cart hit the bridge abutment at about 0.58 seconds with the back wheels experiencing an increase in wheel-rail contact force immediately afterwards. The increase in front wheel load is due to force required to accelerate the wheels and axle upward as the cart travels from the softer approach track (lower wheel elevation) to the stiffer bridge track (higher wheel elevation). The increase of back wheel load is a reaction from upward acceleration of the front axle causing the cart to tilt and consequentially create a downward acceleration of the back axle. The back wheels then experience an increase of load by about 20% to 25% in the transition zone, which is similar to previous analyses (Nicks, 2009). Because of the 9.33 foot distance between the front and back cart axles, the impact occurs about 10 feet or 5 ties away from the bridge abutment.

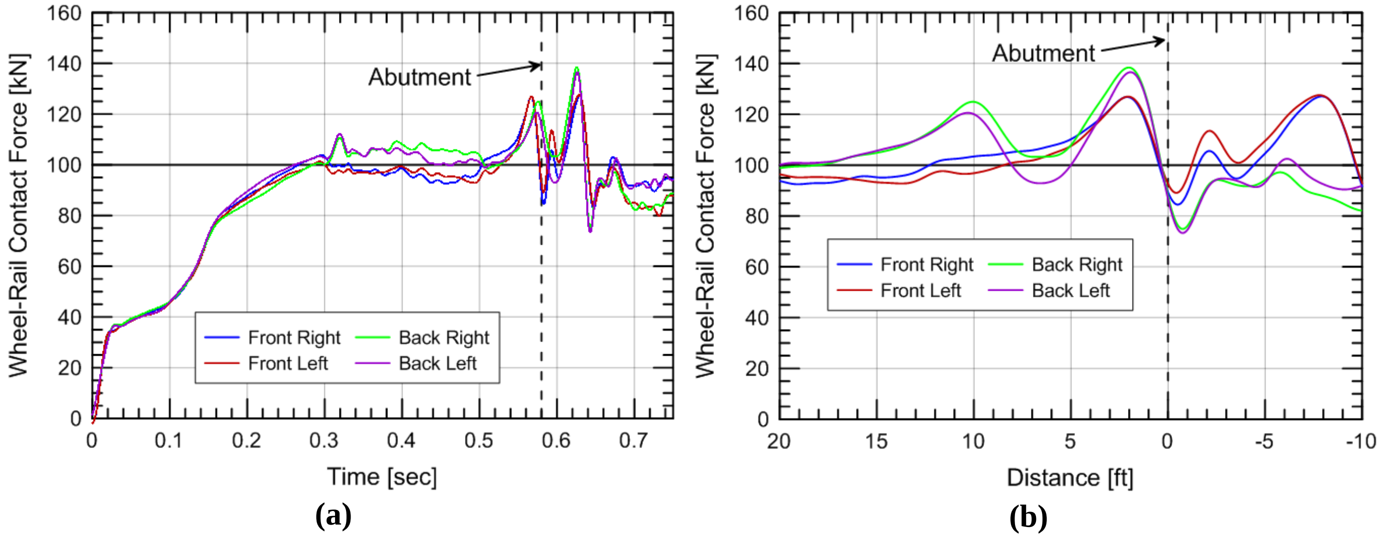

Figure 9: Wheel-Rail Contact Force Time Histories with (a) time and (b) distance.

Figure 10 shows the tie-ballast contact forces with time. Prior to the front wheels passing the bridge abutment, the tie-ballast contact force measured at about 80 kN which equates to a percent wheel load of about 40%. Because the impact of the back wheel occurs about 10 feet or 5 ties from the bridge abutment, Ties T6 and T5 experience load increases of 14% and 22% respectively (tie-ballast contact force of 91 and 98 kN).

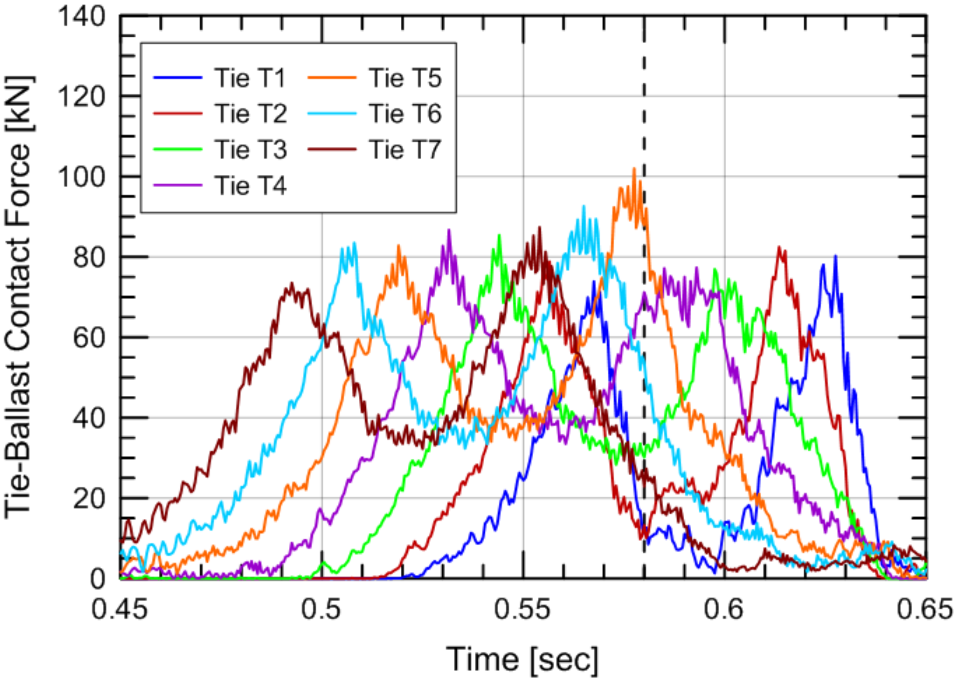

Figure 10: Tie-Ballast Contact Force Time Histories for Ties T1 through T7.

<u>Bridge Approach – Parametric Analysis #1</u>

The first bridge approach parametric analysis with tie-ballast gaps involves a situation where the tie-ballast gaps incrementally increase in height as the bridge abutment is reached. This is illustrated in Figure 11 with Ties T1 and T2 having gap heights of 4x, Ties T3 and T4 having gap heights of 2x, and Ties 5 and 6 having gap heights of x. The gap height "x" is varied from 0 – 2 mm. This means the gap height of Tie T1 equals 8 mm when Tie T5 is set to 2 mm.

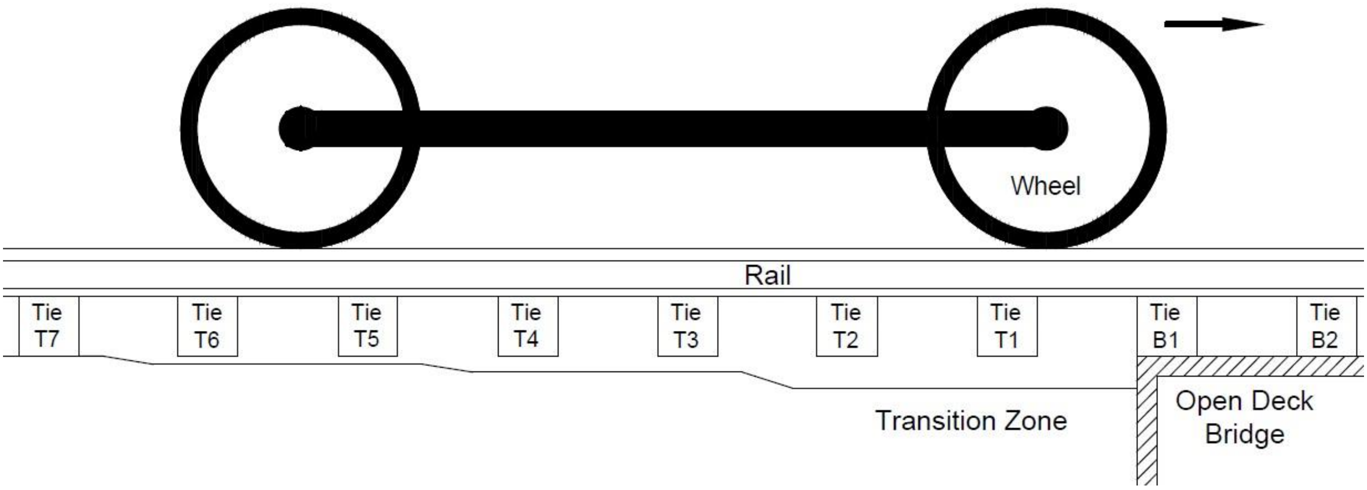

Figure 11: Layout of Tie-Ballast Gaps for Bridge Approach Parametric Analysis #1.

Figure 12 displays the change in maximum percent wheel load from the back wheels due to variation of gap height "x". For a gap height "x" of zero, all ties experience maximum wheel loads of about 30 – 50% (see Figure 10). As the gap height increases, the maximum percent wheel load experienced by Ties T5 and T7 increase as well. At a gap height of 2.0 mm the maximum percent wheel load of Tie T5 approaches 120%, an increase of 146%. All other ties experience a decrease in maximum percent wheel load with increasing gap height as the load is being distributed away from these ties. This behaviour agrees with the open track behaviour because the load is still being redistributed from poorly supported ties to more well supported ties. For example, due to the good tie support at Tie T7, the rail bending causes Tie T5 to contact the ballast prior to Tie T6 and therefore Tie T5 experiences a greater load. This same concept explains why Tie T3 experiences greater load than T4. Additionally, the combination of the back wheel impacting the rail and Tie T5 impacting the ballast amplifies the tie-ballast contact force at T5. The increase in load at Ties T5 and T7 explain how the presence of tie-ballast gaps initially located in the first three or so meters (10 ft.) of the transition zone region can result in tie-ballast gaps spreading outwards from subsequent train passes.

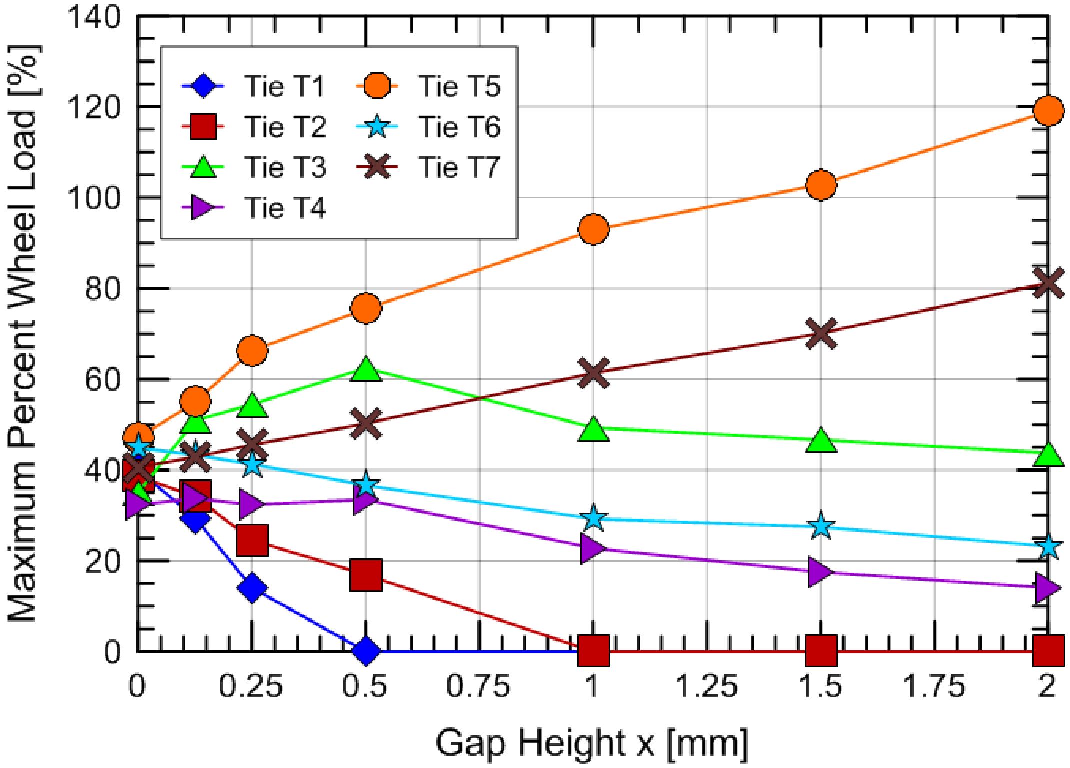

Figure 12: Maximum percent wheel load (maximum tie-ballast contact force / wheel load) for Ties T1 through T7 for Parametric Analysis #1.

<u>Bridge Approach – Parametric Analysis #2</u>
The second bridge approach parametric analysis involves a situation where a single tie (Tie T3) is in intimate contact with the ballast while the remaining ties have tie-ballast gaps of gap height “x” (see Figure 13). This situation has been observed by the authors and appears because the substructure settlement in the bridge approach will not be homogenous, meaning the substructure will settle more in certain locations than others. This may be due to increased loads, uneven ballast compaction, rail joints, fouling, or drainage concerns.

Figure 14 displays the change in maximum percent wheel load due to the variation of gap height “x”. As the gap height increases, the maximum percent wheel load experienced by Tie T3 significantly increases. At a gap height of 2.0 mm, the maximum percent wheel load of Tie T3 surpasses 122%. This represents an increase in tie-ballast load of about 205%. Additionally, Tie T7 shows a slight increase in tie load while Ties T1, T2, T4, T5, and T6 show decreases. This is due to the redistribution of wheel load away from poorly supported ties to the supported ties.

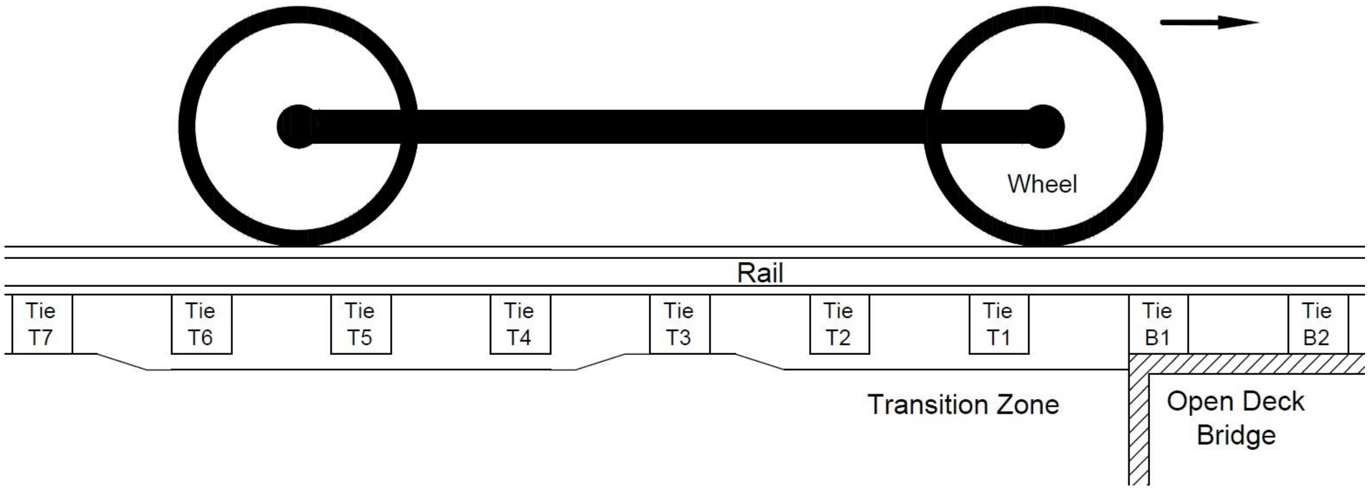

Figure 13: Layout of Tie-Ballast Gaps for Bridge Approach Parametric Analysis #2.

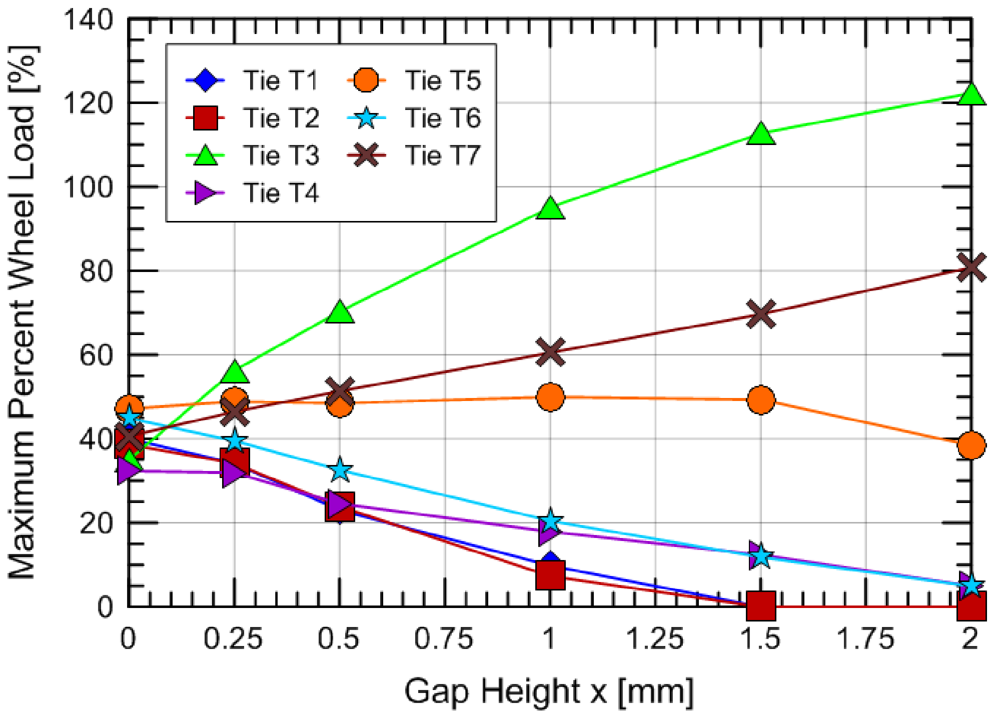

Figure 14: Maximum percent wheel load (maximum tie-ballast contact force / wheel load) for Ties T1 through T7 for Parametric Analysis #2.

These analyses show that uneven settlement in the transition zone substructure can significantly increase tie-ballast loads of the most well supported ties in the transition zone. This means the wheel loads will not be evenly distributed amongst the ties in the transition zone but the majority of wheel load will concentrate on one or a few ties. This overloads the well supported ties resulting in increased settlement of the underlying substructure or even damage to those ties. Subsequently, the load will then concentrate to the next most "well supported" tie and the cycle continues. Additional rail movement from rail joints can exacerbate the problem, which is not currently included in the numerical model.

SUMMARY

This paper presents a numerical model to simulate the passing of a bogie over both open track and bridge approach situations. By creating a physical discontinuity between the tie and ballast, the effect of tie-ballast gaps is investigated on the tie-ballast loads. The following summarizes the main findings of the study:

- In open track scenarios, the inclusion of tie-ballast gaps redistributes the wheel load away from the tie with a tie-ballast gap onto more well supported ties. In the case of 1 mm gap at a single tie, all the wheel load is redistributed to the adjacent ties. In the case of a single well-supported tie surrounded by ties with 1 mm gaps, the well supported tie experiences an 83% increase in load. For situations with varying gap heights, the load redistribution is highly sensitive and changes significantly depending the on the tie-ballast gap height of the tie in interest and the surrounding ties.

- In bridge approach scenarios with no tie-ballast gaps, the change in wheel elevation as the train passes the bridge abutment produces a 25% increase in wheel load within the transition zone region. This help explains why the substructure settlement in transition zones tends to be higher than the surrounding track.

- When tie-ballast gaps are introduced into the bridge approach model, redistribution of wheel loads and impact forces can increase the tie-ballast loads by almost 200% at tie-ballast gap heights of 2.0 mm. This means that wheel load is often not evenly distributed amongst the ties in the transition zone but distributed so certain ties experience greater load while other ties will experience lesser amounts. Because transition zone substructures typically settle unevenly, these results suggest that designing track to experience only 40% of the static wheel load may underestimate the physical loading conditions.

This numerical model will continue to be used to simulate various tie-ballast gap scenarios to better understand the long term behavior of bridge transition zones.

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