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IMPORTANCE OF SIDE RESISTANCE IN 3D STABILITY ANALYSIS

Kamran Akhtar[1] and Timothy D. Stark[2]

ABSTRACT

Present 3D limit equilibrium (LE) methods do not incorporate shear resistance from near vertical sides parallel to the direction of slide movement. Consequently, the computed 3D factor of safety (FS) is underestimated and overestimates shear strength parameters from inverse analyses. Different techniques have been proposed to incorporate side shear resistance including using an at-rest earth pressure (K_O) and Mohr-Coulomb strength criteria to estimate the shear resistance. The present study uses continuum mechanics to calculate the magnitude of side shear resistance along near vertical sides of a translational slide mass. Results of the parametric study show use of an earth pressure coefficient (K_τ) that is in-between at-rest (K_O) and active (K_A) earth pressure and Mohr-Colombo strength criteria provides a reasonable estimate of the side shear resistance and 3D/2D FS ratios that are in agreement with finite element (FE) and finite difference (FD) a continuum analyses. Based on these findings, charts showing the influence of shear resistance on 3D/2D FS ratios for various slope inclinations and geometries are presented herein.

Introduction

Two-Dimensional (2D) limit equilibrium (LE) analyses are based on a plane strain condition that assumes the slide mass or cross-sections, is infinite in the direction perpendicular to slide movement and therefore 3D effects (end effects) are negligible compared to the shear resistance mobilized along the failure surface. This assumption is acceptable if the width of the slide mass is large compared to its height, i.e., ratio of width (W) to height (H) of the slide mass is greater than four (Arellano and Stark, 2000). However, most, if not all, landslides are not infinitely long and vary perpendicular to slide movement. Therefore, application of a 2D analysis to a 3D problem is not accurate but believed to be conservative/sufficient for engineering purposes because the end effects are neglected. Past research, e.g., Hutchinson and Sarma 1985; Cavounidis 1987; Hungr 1987; Duncan 1996, shows that 3D analyses yield greater FS values than those calculated using 2D analyses for the critical failure surface, all other things being equal. 2D analyses are conservative because the resistances along the out-of-plane faces of the slide mass are neglected in the analysis. This conservatism may be acceptable for slope designs but in the case of inverse analyses of landslides, 2D analyses may result in unconservative values of back-calculated shear strength by as much as 30% (Stark and Eid 1998). 3D analyses also allow modeling of changes in slope geometry and material properties across the slide mass.

[1] Civil and Environmental Eng., National University of Sciences and Technology (NUST), H-12 Campus, Islamabad, 44000.
[2] Professor of Civil and Environmental Eng., Univ. of Illinois, 205 N. Mathews Ave., Urbana, IL 61801.

For a translational landslide, Stark and Eid (1998) show three-dimensional (3D) LE methods do not incorporate the effects of shear resistance along vertical or near vertical sides of the slide mass parallel to the direction of slide movement. Consequently, the computed 3D factor of safety (FS) is underestimated which results in an overestimate of inverse analysis of shear strength parameters. To overcome this limitation, Stark and Eid (1998), Arellano and Stark (2000), and Eid et al. (2006) suggest different techniques to incorporate the side shear resistance in 3D LE computations. These three techniques estimate the magnitude of side shear force using at-rest earth pressure (K_O) and Mohr-Coulomb strength criteria.

The present study uses finite element (FE) and finite difference (FD) continuum analyses to calculate the magnitude of side shear resistance along vertical or near vertical sides of a translational slide mass. Results of the parametric study show use of K_O for approximating the shear resistance results in an overestimation of the 3D/2D FS ratio estimated using FE and FD analyses. However, use of an earth pressure coefficient (K_τ) that is in-between at-rest (K_O) and active (K_A) earth pressure provides a reasonable estimate of the side shear resistance and 3D/2D FS ratios that are in agreement with FE and FD analyses. Based on these findings, the charts provided by Arellano and Stark (2000) showing the influence of shear resistance on 3D/2D FS ratios for various slope inclinations and geometries are updated herein because they were developed using K_O.

Influence of Side Resistance in 3D Analysis

Stark and Eid (1998) and Arellano and Stark (2000) show translational slides exhibit a significant difference (~40%) between 2D and 3D values of FS. This difference is less pronounced in slopes that fail in a rotational failure mode or have non-vertical sides. End effects are more pronounced in translational than rotational failures for the following reasons (Stark and Eid 1998; Arellano and Stark 2000):

- Slopes failing in translational mode usually involve either a significantly higher or lower mobilized shear strength along the back scarp and sides of the slide mass than along the base. These situations can result in a significant difference between the 2D and 3D FS. This difference is less pronounced in slopes failing in a rotational failure mode because they usually involve more homogeneous materials so the strength difference of the materials involved is lower than translational slides.

- A translational failure can occur in relatively flat slopes because of weak underlying material(s). The flatter the slope, the greater the difference between 2D and 3D FS because of the larger area of the sides of the slide mass.

- A translational failure often involves a long and nearly horizontal failure surface through a weak underlying soil layer.

- A translational failure often involves a drained shearing condition. This facilitates estimation of the mobilized shear strength of the materials involved because shearinduced pore-water pressures do not have to be estimated only hydrostatic pressures.

The above reasons affect the 3D/2D FS ratio for translational slides and therefore should be considered for 2D and 3D slope stability analyses. Also as the inclination of the sides parallel to the direction of motion of the slide mass increases, the shear surface area decreases. Therefore in translational slides vertical sides provide the minimum amount of 3D shear resistance because the effective normal stress acting on these sides is only due to the lateral earth pressure.

Stark and Eid (1998) and Arellano and Stark (2000) used CLARA 2.31 (Hungr 1998) to perform 3D parametric analyses because: (1)user friendly input of slope geometry and pore-water pressure conditions; (2) uses Janbu's (1954) simplified procedure for 2D and 3D analyses, which is suitable for translational mode of failure; (3) external loads may be specified and used to simulate the shear resistance acting on vertical sides; and (4)can perform 2D analysis from a 3D data file.

Based on the results of a parametric study, Stark and Eid (1998) show that all LE software does not consider the shear resistance along the vertical sides of the slide mass. This leads to underestimation of 3D FS, especially when the material along the vertical sides has a greater shear strength than the material along the base of the slide mass.

Consideration of Side Forces in 3D LE Slope Stability Software

In 3D slope stability software, a user defines the grid extent in x and y-directions. The user also specifies the number of rows and columns, which essentially determines the size of the individual vertical 3D columns. These vertical columns are the 3D equivalent of vertical slices in a 2D analysis. Similar to a 2D analysis, the resisting force is computed at the base of each column, instead of vertical slice, using the shear strength of the material through which the column base rests. The resisting forces due to the earth pressure and Mohr-Coulomb shear strength parameters applied to the vertical sides of the columns along the ends of the slide mass are not computed by existing 3D software because only the base is considered. To overcome this limitation, different techniques have been suggested to include the shear resistance along the vertical or near vertical sides of the slide mass, which are briefly described below.

Stark and Eid (1998)

Stark and Eid (1998) suggest using a shear force equal to the side resistance to calculate the 3D FS. This is accomplished by assuming an “imaginary” material layer surrounds the sides of the slide mass not the back scarp. The material properties of the imaginary layer only affect the shear strength along the vertical sides and not the base or the back scarp of the slide mass. The soil parameters of the imaginary layer are:

- Unit weight of the imaginary layer which is equal to that of the upper layer, $\gamma'_{imaginary}=\gamma'_{upper}$

- Imaginary layer is frictionless, $\phi'_{imaginary}=0$

- The cohesion of the imaginary layer is equal to the shear strength due to K_O, acting on the vertical sides of the slide mass, $c'_{imaginary}=K_O\ \sigma'_v \tan \phi'_{upper}$, where, σ'_v is the average vertical effective stress over the depth of the sliding mass side, *and* $K_O = 1-\sin \phi'_{upper}$.

In addition, each vertical side of the sliding mass is assigned a slight (less than 5°) outward inclination to include a single row of columns so the analysis and software can calculate the effect of cohesion in its resisting force calculations.

Arellano and Stark (2000)

Arellano and Stark (2000) use a rectangular slide mass without rounded or a curved head scarp in their parametric study. To include side resistance, an external horizontal and vertical side force equivalent to the shear resistance due to at-rest earth pressure (K_O) acting on the vertical sides at the centroid of the two parallel sides is included. The technique for calculating the shear resistance acting on the vertical sides is the same as used in Stark and Eid (1998), i.e., $c'=K_O\ \sigma'_v \tan\phi'_{upper}$ and $\phi'=0$.

The side shear force, S', acting on the vertical sides is estimated by multiplying c' by the cross-sectional area of the vertical side. For simplicity, the side resistance of only the upper layer is used and the small side area between the interface of upper material and lower material and the base of failure surface is neglected when estimating the cross-section centroid. Additionally, it is assumed that S' acts parallel to the base of the failure surface at a slope of 3% down slope.

Arellano and Stark (2000) investigate effect of side shear resistance on three different slope inclinations, i.e., 1H:1V, 3H:1V, and 5H:1V, to study the ratio of 2D/3D FS. For each slope inclination, W/H ratios of 1, 1.5, 2, 4, 6, 8, and 10 were analyzed. Arellano and Stark (2000) indicate that because the slope model has uniform cross-sections across the slope, the 2D FS at all cross-sections is the same. Additionally, 3D FS is equal to 2D FS if no external loads are applied, which confirms the finding of Stark and Eid (1998) that shearing resistance along the parallel vertical sides of a slide mass are not considered in 3D FS calculations using existing LE slope stability software. After including the shear resistance along vertical sides, Arellano and Stark (2000) present a relationship between 3D/2D FS and W/H ratio for different slope inclinations. For a slope of 1V:1H with a W/H ratio of one, 3D/2D FS ratio is about 1.30 which indicates a 30% increase in 2D FS. On the contrary, a slope of 5V:1H with the same W/H ratio has a 3D/2D FS ratio of about 3.2. This unusually high 3D/2D FS ratio for a 5H:1V slope is due to the use of at-rest earth pressure, instead of an earth pressure between active and at-rest, and is revised in this paper because a 3D/2D FS ratio of 3.2 does not match field observations and FE and FD analyses.

Eid et al. (2006); and Eid (2010)

Eid et al. (2006) and Eid (2010) include the shear resistance along the two vertical sides of a slide mass by imposing a "group" of external horizontal and vertical forces (S_y and S_z) that are the components of the shear resisting force (*S*). Calculation of the resisting force is the same as used by Stark and Eid (1998) and Arellano and Stark (2000) except the forces generated by at-rest earth pressure and pore water pressure are calculated separately and then imposed at the centroids of the their corresponding areas on the vertical sides of slide mass. In Arellano and Stark (2000) the earth pressure forces are approximated using the average vertical effective stress over the depth of the sliding mass and applied at the centroid of the vertical sides of the slide mass.

Magnitude of Side Resistance

To investigate the actual magnitude of side shear resistance to modify existing 3D LE methods, the slide mass model used by Arellano and Stark (2000) was analyzed in 2D and 3D using LE, FE, and FD software. 2D and 3D LE analyses were performed using CLARA-W (Hungr 2001) and a 3D extension of Janbu's (1954) procedure. 2D and 3D FE analyses were performed using PLAXIS 3D Tunnel V.2 (Brinkgreve and Broere 2004). 2D and 3D FD analyses were also performed using FLAC (Itasca 2000) and FLAC3D (Itasca 2002), respectively.

The objectives of these analyses are to: (1) determine the magnitude of 3D/2D FS ratios computed by FE and FD procedures; (2) determine magnitude of 3D/2D FS ratios computed using K_O and K_A for the side shear resistance in 3D LE analyses; and (3) develop recommendations for the coefficient of earth pressure that should be used to estimate the side shear resistance in 3D LE analysis of translational landslides with vertical or near vertical sides.

Parametric Slope Model

As a sequel to Arellano and Stark (2000), Chugh (2003) investigated the 5H:1V slope model using FD analysis using FLAC and shows the significance of boundary conditions in the continuum method. Chugh (2003) reports 3D/2D FS ratio of 2.05 for W/H ratio of one and a 5H:1V slope, which is lower than 3D/2D FS ratio of 3.2 reported by Arellano and Stark (2000).

To draw a direct comparison with previous research, the slope model used by Arellano and Stark (2000) was reanalyzed using LE, FE and FD procedures herein. Detailed information about the model is given in Arellano and Stark (2000). Slope inclinations of 1H:1V, 3H:1V, and 5H:1V with a height (H) of 10 m were analyzed. For FE and FD analyses, each slope inclination was analyzed with W/H ratios of 1, 2, 5, and 10 with friction angles of 30° and 8° for upper and lower materials, respectively, to model a translational sliding situation. LE analyses were performed for W/H ratios of 1, 1.5, 2, 4, 5, 6, 8, and 10 with four combinations of $\phi'_{upper}/\phi'_{lower}$ values. The friction angle of upper material (ϕ'_{upper}) was kept at 30° while the friction angle of the lower material (ϕ'_{lower}) was assigned values of 8°, 10°, 20°, and 30°.

To simulate a natural bedding plane or a weak geosynthetic interface in a landfill liner system, the lower material was assumed to slope at 3% down slope. The groundwater or leachate level was placed at a height of H/2 as measured at a distance L from the toe and linearly decreasing to a height of zero at the toe.

LE, FE, and FD are different procedures and use different solution strategies. In continuum analysis, the slope models were extended past the locations where the slope failure is likely to occur. Therefore, slope models in FE and FD analyses are wider than models used for the same inclination in LE analysis. Also, the lower material is represented by a layer of 0.8 m thick, which is followed by a bottom block. The presence of the bottom block ensures the failure surface remains in the weaker layer to model a translational failure.

For a 3D LE analysis, shear resistance along the parallel sides of the slide mass was incorporated by adding external horizontal and vertical side forces using two separate techniques:-

- Application of one set of external horizontal and vertical side forces at overall centroids of the two parallel sides as done by Arellano and Stark (2000).
- Application of adding maximum possible sets of external horizontal and vertical side forces at the centroids of respective individual of the active columns on the two parallel sides. Maximum number of loads that can be specified in CLARA-W is 100, i.e., 50 sets (horizontal and vertical) of loads may be applied on either parallel sides of slope model.

This 3D slope models used in the FE and FD analyses include 6m wide end blocks and displacement condition of fully fixed (u=0, v=0, and w=0) at the boundaries. In addition, the 3D analysis in FLAC3D uses side blocks with higher strength and an interface between the slope and end blocks to allow relative movement at the slope-block contact. The material properties used in the CLARA-W, PLAXIS, and FLAC analyses are shown in Table 1.

Effect of Shear Resistance Along Vertical Sides

To verify the magnitude of shear resistance along the vertical sides of the slide mass, 3D/2D FS ratios obtained from FE and FD analysis for three slope inclinations are plotted for different W/H ratios. For the 3D LE analysis, horizontal and vertical side force equivalents were computed using the shear resistance due to at-rest earth pressure ($K_O=1-\sin\phi'$), active earth pressure ($K_A=1-$

sinϕ'/1+sinϕ'), and an earth pressure coefficient (K_τ) that is in-between the K_O and K_A values. For simplicity/consistency in the analyses, a value of $K_\tau=0.5*(K_O+K_A)$ was used for the in-between case or the average between K_O and K_A. Thereafter, 3D/2D FS ratios computed for the LE analysis were compared with results of FE and FD analysis to determine the optimal earth pressure coefficient to use to incorporate 3D side resistance in LE analyses.

Table 1. Material Properties for Stability Analyses of Slope Model.

Parameter	**Upper Material**	**Lower Material**	**Bottom Block**	**End Blocks[2]**	**Interface[3]**
Unit weight[1], γ (kN/m^3)	17	18	18	25	-
Cohesion, c' (kP a)	0	0	0	0	0.05
Friction angle, ϕ' (◦)	30	8,10,20,30	40	45	30
Dilatation angle, ψ (◦)	0	0	0	0	-
Young's modulus (kN/m^2)	3×10^4	3×10^3	3×10^5	3×10^6	-
Poisson's ratio, ν	0.35	0.35	0.35	0.35	-
Bulk Modulus (kN/m^2)	3×10^4	3×10^3	3×10^5	3×10^6	-
Shear Modulus (kN/m^2)	1×10^4	1×10^3	1×10^5	1×10^6	-
Normal stiffness (kN/m^2)	-	-	-	-	1×10^4
Shear stiffness (kN/m^2)	-	-	-	-	1×10^3

[1] Density ρ(k g/m^3) = Unit weight x 1000/9.81;
[2] End blocks in PLAXIS analysis use same material properties as slope;
[3] Only used in FLAC analysis

Results from Continuum Analysis

The FE and FD analyses shows that 3D/2D FS ratios for all W/H combinations are greater than unity, i.e., 3D FS is always greater than 2D FS. The highest value of 3D/2D FS ratio corresponds to the 5H:1V slope for W/H=1. The highest values of 3D/2D FS ratio is 2.04 and 2.05 obtained from FE and FD analyses, respectively instead of 3.2 as reported by Arellano and Stark (2000). The values of 2.04 and 2.05 are in agreement with 2.05 reported by Chugh (2003).

In summary, 3D/2D FS ratios from continuum procedures (FE and FD) show similar trends. For example, 3D/2D FS ratios increase with decreasing W/H ratios and for a given W/H ratio and flatter slopes have higher 3D/2D FS ratios. These observations are in accordance with the findings reported by Arellano and Stark (2000). 3D/2D FS ratios obtained from the FD analysis are slightly higher than FE. Therefore, the FD and FE analyses are used as upper and lower bounds, respectively, for 3D/2D FS ratios for each slope inclination of model geometry.

Results from LE Analysis

For illustration, 3D/2D FS values for 5H:1V slope obtained from FE, FD, and LE using external side forces estimated using K_O, K_A and K_τ (applied at overall centroids of the two parallel sides) are compared in Figure 1.

Figure 1 shows using K_O produces 3D/2D FS ratios that are much greater than those produced by K_A. This is caused by K_O being almost 50% greater than K_A, i.e., for a ϕ'_{upper}=30°, K_O=0.5 and K_A=0.33, and the resulting side resistance being included via the cohesion parameter in the FS calculation. Figure 1 also shows that using K_O to estimate side shear resistance results in 3D/2D FS ratios that are higher than the upper limit set by FD analysis for the same slope inclination. The 3D/2D FS ratio for the 5H:1V slope and W/H=1 is the highest. This ratio (3D/2D FS=3.2) is significantly higher than the FD 3D/2D ratio because K_O produces too high of a side resistance. However, using K_A in all three slope inclinations underestimates 3D/2D FS ratios for W/H ratios less than 2. Therefore, the optimal earth pressure to estimate field side resistance appears to be in-between K_O and K_A.

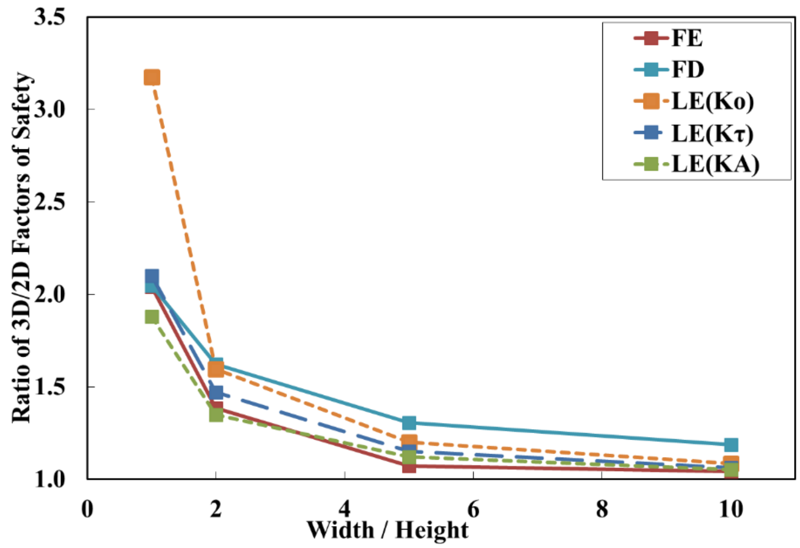

Figure 1. Comparison of 3D/2D FS ratios from FD, FE, and LE (with side forces) for 5H:1V slope and various W/H ratios.

In summary, for all three slope inclination studied, 3D/2D FS ratios are within the upper and lower FD and FE bounds when an earth pressure coefficient (K_τ) equal to the average of K_O and K_A is used.

The use of an intermediate value of K_τ for calculation of side forces is supported by field slide mass observations where, generally the slide mass is cracked near the ground surface and the cracks decrease in width with depth. Therefore, near the ground surface the side resistance may agree better with K_A and near the base of the slide mass it may agree better with K_O. Based on triaxial compression tests, Lambe and Whitman (1969) report horizontal strain of less than 0.5% is required to change the stresses from at-rest to active earth pressure. Therefore, it is possible that after the slip surface develops and movement begins, the at-rest earth pressure transitions to an active pressure.

It is probably confusing and illogical that an earth pressure applied to the side of the slide mass is being used to estimate the side shear resistance when the slide mass is moving perpendicular to the lateral earth pressure. The cracks mentioned above near the ground surface are not in the direction of slide movement so do not appear related to lateral earth pressure. However, the earth pressure coefficient is only being used to develop a reasonable approximation of the side shear resistance and not to explain the lateral pressure applied to the slide mass because the slide mass eventually moves perpendicular to the applied earth pressure. The lateral earth pressure is simply being used as a familiar means for estimating the horizontal stress applied to the side of a slide mass which is used to estimate an empirical shear resistance to approximate the side shear resistance.

The effect of point(s) of application of external horizontal and vertical side forces at the overall centroids of two parallel sides or centroids of individual active columns is shown in Figure 2. Dotted lines show the 3D/2D FS ratios obtained by applying side shear forces at overall centroids of the two parallel sides, while solid lines show 3D/2D FS ratios which were obtained by applying side shear forces at maximum number of active columns on the two parallel sides. The results show the variation in FS computations is about 5% which is less than the acceptable error of 12% reported by Duncan (1996). Therefore, application of one set of external horizontal and vertical side forces at overall centroids of the two parallel sides as done by Arellano and Stark (2000) is considered sufficient for practical application.

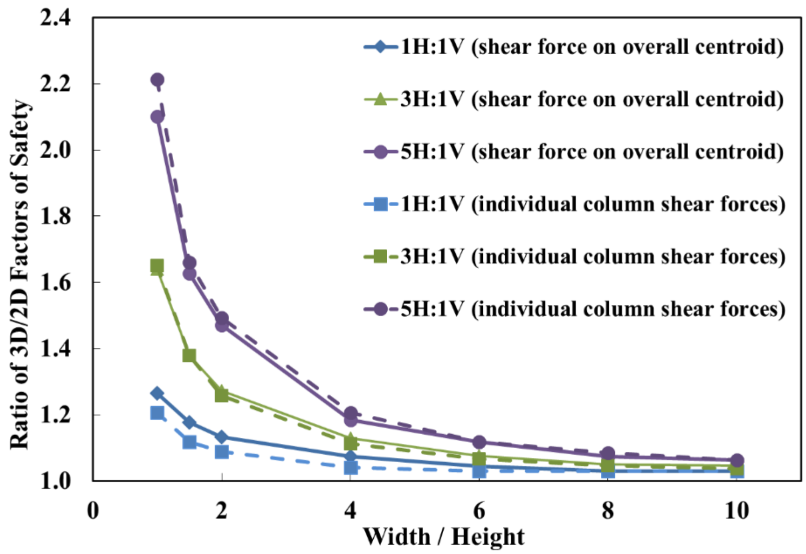

Figure 2. Effect of single or multiple points of application of side shear resistance.

Figure 3 presents a relationship between the 3D/2D FS ratios and W/H for the three slope inclinations considered in the parametric study. Dotted lines show the 3D/2D FS ratios obtained using side shear resistance estimated using K_O, while the solid lines represent 3D/2D FS ratios obtained using K_τ, i.e., in-between K_O and K_A. Figure 3 shows for the 1H:1V slope there is little difference between 3D/2D FS ratios obtained using K_τ or K_O. However, the difference in 3D/2D FS ratios is greater for flatter slopes, i.e., 5H:1V slope (W/H=1), using K_τ and K_O (2.10 and 3.2, respectively). The maximum value of 3D/2D FS obtained using K_τ to calculate side resistance is about 2% higher than 3D/2D FS ratio obtained from FE and FD analyses, i.e 2.05 vs 2.10.

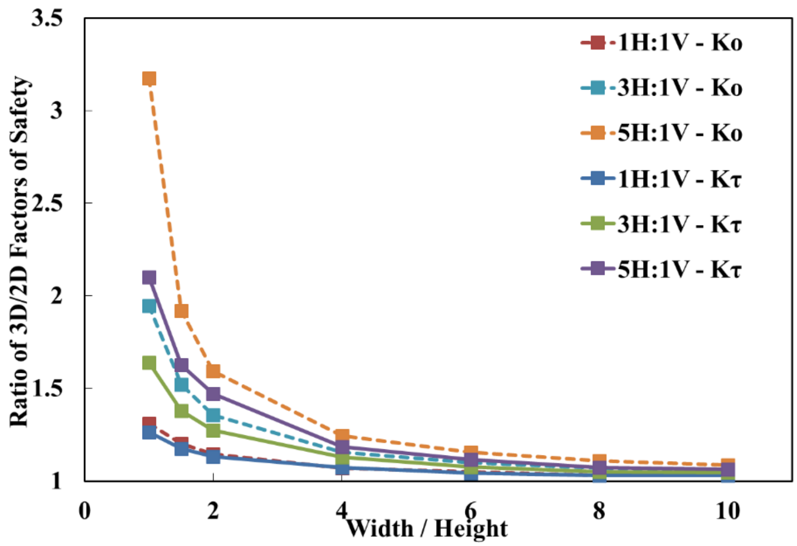

Figure 3. Effect of shear resistance along vertical sides of slide mass using K_O and K_τ procedure

Based on these analyses and K_τ, updated relationships between ratio of 3D/2D FS and W/H for the three slope inclinations are presented in Figure 4. These relationships are for friction angles of 30° and 8° for the upper and lower materials, respectively, and supercede the relationships presented in Arellano and Stark (2000).

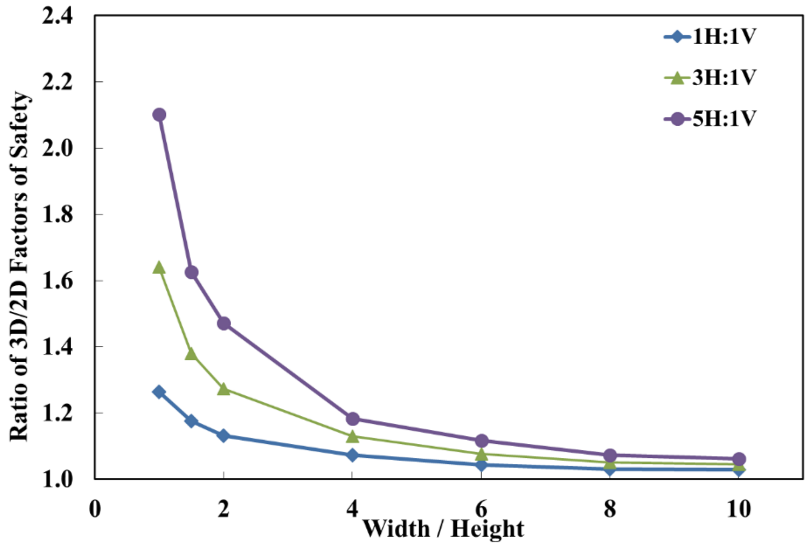

Figure 4. Effect of shear resistance along vertical sides of slide mass using K_τ.

Conclusions and Recommendations

The following conclusions and recommendation are presented based on the FE, FE, and LE analyses presented above:

- 3D/2D FS ratios for all slope inclinations analyzed using FE, FD, and LE procedures are greater than unity.
- To incorporate the shear resistance along the two vertical sides that parallel the direction of movement in translational slides, external forces can be applied in LE software while calculating 3D FS.
- For translational slides with vertical or near vertical sides, continuum procedures (FE and FD) assist in providing better estimates of 3D/2D FS ratios. However, meaningful analyses using continuum mechanics require special attention to slide mass boundary conditions.
- Comparison with FD and FE analyses indicate using K_O to estimate an empirical shear resistance along two vertical sides in LE method overestimates 3D/2D FS ratios for flat (5H:1V) slopes .
- An earth pressure coefficient (K_τ) equal to the average of K_O and K_A provides a better estimate of shear resistance acting along two vertical sides and results in 3D/2D FS ratios that are in agreement with FE and FD analyses.
- For practical purposes, application of one set of external horizontal and vertical side forces at overall centroids of the two parallel sides as done by Arellano and Stark (2000) provides reasonable accuracy in LE analyses.
- The lateral earth pressure is only being used to develop an empirical estimate of the side shear resistance along a slide mass and not to explain the lateral pressure applied to the slide mass. The applied earth pressure is really not applicable to the side shear resistance

because the resistance is developed along the sides of the slide mass as it moves perpendicular to the applied earth pressure.

- LE, FE, and FD analyses show that side resistance is a function of W/H and slope inclination. For example, for a given slope inclination, 3D effects are higher for lower W/H ratios, which tends to become constant for greater W/H ratios (e.g., W/H>6).
- Flatter slopes have higher 3D/2D FS values because of larger side area.
- Maximum 3D/2D FS ratio obtained from continuum analyses is 2.05 (5H:1V slope and W/H=1).
- 3D values of FS should not be compared to 2D regulatory value of FS, e.g., 1.5, because of lower safety due to inclusion of the side resistance. A FS greater than 1.5 should be used to create the same level of safety.

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