EMPIRICAL CORRELATIONS - DRAINED SHEAR STRENGTH FOR SLOPE STABILITY ANALYSES

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ABSTRACT: Empirical correlations provide estimates of parameter values for preliminary design, verification of laboratory shear test data, and confirmation of back-analysis of a failed slope. The empirical correlations presented herein use liquid limit, clay-size fraction, and effective normal stress to capture the variability and stress dependent nature of drained residual and fully softened strength envelopes. This paper describes the testing and analysis used to increase the number of data points in the existing correlations, expand the residual strength correlation to include an effective normal stress of 50 kPa, and develop correlations between values of liquid limit and clay-size fraction measured using sample processed through a Number 40 sieve (ASTM procedure) material and values derived using ball milled/disaggregated sample. In addition, equations are presented to express the empirical correlations which were used to develop a spreadsheet that estimates the residual and fully softened friction angles based on entered values of liquid limit and clay-size fraction.

Keywords: shear strength, residual friction angle, fully softened friction angle, empirical correlation, clay, shale

INTRODUCTION

Different empirical correlations, e.g., Skempton (1964), Lupini et al. (1981), Mitchell (1993), and Terzaghi et al. (1996), for drained residual and fully softened shear strengths have been proposed by considering these strengths as a function of a single or combination of parameters such as, clay-size fraction (CF), plasticity index (PI), and liquid limit (LL). Residual shear strength is primarily dependent on mineral composition, which is related to plasticity and grain size characteristics. Fully softened strength corresponds to random arrangements of clay particles and is numerically equivalent to the drained peak strength of a normally consolidated specimen (Skempton, 1970). Therefore, particle size, shape, interlocking, and degree of orientation are important in estimating the fully softened shear strength of clays. Both residual and fully softened strengths are stress dependent (Chandler, 1977, Lupini et al., 1981, Stark and Eid, 1994 and 1997, Mesri and Shahien, 2003, and Stark et al., 2005). Therefore, an empirical correlation incorporating effective normal stress, LL, and CF, as suggested by Stark and Eid (1994 and 1997) and Stark et al. (2005), provides a good estimate of the friction angles. These empirical correlations were developed using torsional ring shear test results and verified using back-analysis of landslide case histories.

This paper describes the testing and analysis used to increase the number of data points in the correlations suggested by Stark and Eid (1994 and 1997) and Stark et al. (2005) and also the effect of sample preparation on LL, CF, effective stress residual secant friction angle (ϕ'_r), and fully softened friction angle (ϕ'_{fs}). Additional improvements in the empirical correlations for drained ϕ'_r and ϕ'_{fs} include expanding the residual strength correlation to include an effective normal stress of 50 kPa, developing mathematical equations for each trend line in the three CF groups and each effective normal stress, and providing recommendations for use in stability analyses. The resulting mathematical equations were used to develop a spreadsheet for estimating values of drained residual and fully softened friction angles.

A stress dependent strength envelope is recommended for analysis of slopes and back-analysis of landslides (Stark and Eid, 1994 and 1997) and exhibits maximum curvature or stress dependency at low effective normal stresses, i.e., effective normal stresses of less than 100 kPa. As a result, it is desirable for the estimated strength envelope to include an effective normal stress of 50 kPa. To accomplish this, the empirical correlation for drained residual friction angle suggested by Stark et al. (2005) is extended herein to include an effective normal stress of 50 kPa with new data and a normal stress trend line for 50 kPa for all three CF groups was developed.

EMPIRICAL CORRELATION FOR DRAINED RESIDUAL SECANT FRICTION ANGLE OF FINEGRAINED SOILS

Many researchers, e.g., Skempton (1964), Voight (1973), Kanji (1974), Seycek (1978), Lupini et al. (1981), Skempton (1985), Mesri and Cepeda-Diaz (1986), Collotta et al. (1989), Stark and Eid (1994 and 1997), Mesri and Shahien (2003), Wesley (2003), and Tiwari and Marui (2005), have proposed empirical correlations for drained residual friction angles using CF, LL, and/or plasticity index (PI) of clays or some other parameter based on CF, PI, Activity, or LL. Stark and Eid (1994) present ring shear drained residual friction angles as a function of LL and incorporate the effect of CF and effective normal stress in a single correlation. The empirical correlation was developed using ring shear test results on thirty-two natural clay soils. Stark et al. (2005) refine the ϕ'_r empirical correlation proposed by Stark and Eid (1994) by adding test results for an additional thirty-four soils for a total of sixty-six natural soils. The empirical correlation proposed by Stark and Eid (1994) and refined by Stark et al. (2005) reduced the scatter as compared to other empirical correlations. The scatter for each CF group of Stark et al. (2005) and the correlations presented herein is about 3-4 degrees.

Updated Empirical Correlation for Drained Residual Secant Friction Angle

The empirical correlation for ϕ'_r suggested by Stark et al. (2005) incorporates the three main factors, clay mineralogy (LL), amount of clay mineral (CF), and effective normal stress (σ'_n), that influence the residual strength which helps to explain reduction in scatter. The correct estimate of values of LL and CF measured using disaggregated samples discussed below, can assist in estimating a reliable value of ϕ'_r. Thus, the correlation suggested by Stark et al. (2005) has been used to provide estimates of ϕ'_r for use in preliminary design, verification of data obtained from laboratory tests, and confirmation of back-analysis of failed slopes.

The empirical correlation uses three different CF groups, i.e., CF $\leq$ 20%, 25% $\leq$ CF $\leq$ 45%, and CF $\geq$ 50%, to account for three different shearing behaviors, i.e., rolling, transitional, and sliding, respectively, as suggested by Lupini et al. (1981) and Skempton (1985). Values of LL and CF can be used to estimate ϕ'_r for various effective normal stresses to develop a stress dependent residual strength envelope. This stress dependent strength envelope should be used directly in stability analyses of preexisting landslides instead of a friction angle and/or cohesion value. Because a stress dependent residual strength envelope has more curvature at low values of σ'_n, data for σ'_n < 100 kPa were developed and added to the empirical correlation herein.

Inclusion of σ'_n = 50 kPa in Correlation for Drained Residual Secant Friction Angle

The nonlinear residual strength envelope is most pronounced, i.e., has greatest curvature, at low effective normal stresses, e.g., σ'_n < 100 kPa, and it becomes more linear at higher effective normal stresses (see ring shear data in Fig. 1). Stark and Eid (1994) and Stark et al. (2005) present a relationship between LL and ϕ'_r for values of σ'_n of 100, 400, and 700 kPa for three CF groups. However, Stark and Eid (1994) and Stark et al. (2005) do not present any data and/or a trend line for σ'_n < 100 kPa. To capture this nonlinearity, a trend line for σ'_n = 50 kPa was developed for each CF group so a well-defined stress dependent residual strength envelope could be estimated using residual secant friction angles for values of σ'_n of 0, 50, 100, 400, and 700 kPa.

During the present study, torsional ring shear data from Eid (1996), data generated by Stark et al. (2005), and testing of seven additional soils herein following ASTM D 6467 (ASTM, 2010a) were used to develop the trend line for σ'_n = 50 kPa. This brings the total number of natural soils used to create the correlation shown in Fig. 2 to seventy-three (73). The empirical correlation in Fig. 2 can be used to estimate the ϕ'_r values for σ'_n of 0, 50, 100, 400, and 700 kPa using CF and LL of a soil. The estimated ϕ'_r value for each value of σ'_n can be used to calculate the residual shear stress (τ_r) which can be used to plot the drained residual strength envelope using the origin. The stress dependent strength envelope developed from the five values of σ'_n (0, 50, 100, 400, and 700 kPa) can be used directly in the stability analysis of preexisting landslides or slopes that may undergo shear movement.

In summary, the addition of data and a trend line for σ'_n = 50 kPa in the empirical correlation shown in Fig. 2 provides a better estimate of the complete stress dependent residual strength envelope than prior correlations for use in stability analyses (see Fig. 1).

Equations for Updated Empirical Correlations for Drained Residual Secant Friction Angle

Use of the correlation between LL and ϕ'_r in Fig. 2 requires the user to have the figure available to obtain values of ϕ'_r for a given LL and CF. Having the empirical correlation available only in graphical form also made it difficult to incorporate the correlation in slope stability software and continuum methods. As a result, the present study developed a separate mathematical equation for each σ'_n trend line of the proposed correlation which can be used to estimate ϕ'_r using LL and the CF group. The value of CF determines the required equation to be used, so the LL value is the only input parameter used in the equation to estimate ϕ'_r for various values of σ'_n.

The mathematical expressions developed herein are in excellent agreement with the trend lines, not the data, suggested by Stark et al. (2005). The empirical correlation for ϕ'_r of CF Group #1 and for LL values ranging from 30% to less than 80% (30% $\leq$ LL < 80%) are shown as Equations (1.1) to (1.4) below. The upper bound for LL is specified because no ring shear data are available outside of this LL range. The ring shear data along with the trend lines sketched by Stark et al. (2005) for CF Group #1 and the trend lines sketched from the newly developed Equations (1.1) to (1.4) are compared in Fig. 3. Fig. 3 shows excellent agreement between the trend lines suggested in Fig. 2 and Equations (1.1) to (1.4). Thus, a second degree polynomial can adequately represent the trend lines for CF Group #1 for all four effective normal stresses.

$$\left(\phi_r\right)_{\sigma'_n=50\text{kPa}} = 39.71 - 0.29(\text{LL}) + 6.63\text{x}10^{-4}(\text{LL})^2 \qquad (1.1)$$

$$\left(\phi_r\right)_{\sigma_n^{'}=100\text{kPa}} = 39.41 - 0.298(\text{LL}) + 6.81\text{x}10^{-4}(\text{LL})^2 \quad (1.2)$$

$$\left(\phi_r\right)_{\sigma_n^{'}=400\text{kPa}} = 40.24 - 0.375(\text{LL}) + 1.36\text{x}10^{-3}(\text{LL})^2 \quad (1.3)$$

$$\left(\phi_r\right)_{\sigma_n^{'}=700\text{kPa}} = 40.34 - 0.412(\text{LL}) + 1.683\text{x}10^{-3}(\text{LL})^2 \quad (1.4)$$

Another set of equations was developed for the trend lines in CF Group #2 (25%≤CF≤45%) and LL values ranging from 30% to less than 130% (30% ≤ LL < 130%) and are given below in Equations (2.1) to (2.4). Again the upper bound for LL is specified because ring shear data are available only for this specific LL range. A third degree polynomial was used to obtain agreement between the trend lines for CF Group #2 and the mathematical expressions for all four effective normal stresses.

$$\left(\phi_r\right)_{\sigma_n^{'}=50\text{kPa}} = 31.4 - 6.79\text{x}10^{-3}(\text{LL}) - 3.616\text{x}10^{-3}(\text{LL})^2 + 1.864\text{x}10^{-5}(\text{LL})^3 \quad (2.1)$$

$$\left(\phi_r\right)_{\sigma_n^{'}=100\text{kPa}} = 29.8 - 3.627\text{x}10^{-4}(\text{LL}) - 3.584\text{x}10^{-3}(\text{LL})^2 + 1.854\text{x}10^{-5}(\text{LL})^3 \quad (2.2)$$

$$\left(\phi_r\right)_{\sigma_n^{'}=400\text{kPa}} = 28.4 - 5.622\text{x}10^{-2}(\text{LL}) - 2.952\text{x}10^{-3}(\text{LL})^2 + 1.721\text{x}10^{-5}(\text{LL})^3 \quad (2.3)$$

$$\left(\phi_r\right)_{\sigma_n^{'}=700\text{kPa}} = 28.05 - 0.2083(\text{LL}) - 8.183\text{x}10^{-4}(\text{LL})^2 + 9.372\text{x}10^{-6}(\text{LL})^3 \quad (2.4)$$

The trend lines in CF Group #3, i.e., CF ≥ 50%, are divided into two parts to ensure the mathematical expressions are in agreement with the trend lines in Fig. 2. Two equations are required to capture the complicated shape of the Group #3 trend lines. Fig. 2 shows that the left portion of each trend line, i.e., LL < 120%, has significant curvature so it is represented by a polynomial expression. The right portion of the trend line, i.e., LL ≥ 120%, is represented by a linear relationship. This necessitated using separate equations for LL values ranging between 40% and less than 120% and LL values ranging between 120% and 300%. The upper and lower bounds for LL values are specified because of the availability of ring shear test data in this range.

As shown in Equations (3.1) to (4.4), a third degree polynomial represents the trend lines for CF Group #3 and for all four effective normal stresses and for 30% ≤ LL < 120% and the trend lines for CF Group #3 and 120% $\leq$ LL $\leq$ 300% can be represented using a linear relationship (straight line).

$$\left(\phi_r\right)_{\sigma_n'=50kPa} = 33.5 - 0.31(LL) + 3.9x10^{-4}(LL)^2 + 4.4x10^{-6}(LL)^3 \qquad (3.1)$$

$$\left(\phi_r\right)_{\sigma_n'=100kPa} = 30.7 - 0.2504(LL) - 4.2053x10^{-4}(LL)^2 + 8.0479x10^{-6}(LL)^3 \qquad (3.2)$$

$$\left(\phi_r\right)_{\sigma_n'=400kPa} = 29.42 - 0.2621(LL) - 4.011x10^{-4}(LL)^2 + 8.718x10^{-6}(LL)^3 \qquad (3.3)$$

$$\left(\phi_r\right)_{\sigma_n'=700kPa} = 27.7 - 0.3233(LL) + 2.896x10^{-4}(LL)^2 + 7.1131x10^{-6}(LL)^3 \qquad (3.4)$$

$$\left(\phi_r\right)_{\sigma_n'=50kPa} = 12.03 - 0.0215(LL) \qquad (4.1)$$

$$\left(\phi_r\right)_{\sigma_n'=100kPa} = 10.64 - 0.0183(LL) \qquad (4.2)$$

$$\left(\phi_r\right)_{\sigma_n'=400kPa} = 8.32 - 0.0114(LL) \qquad (4.3)$$

$$\left(\phi_r\right)_{\sigma_n'=700kPa} = 5.84 - 0.0049(LL) \qquad (4.4)$$

Drained Residual Strength Envelope

The empirical correlation in Fig. 2 results in a cohesion intercept of zero for the drained residual strength envelope. By definition the residual strength condition results from the reorientation of platy clay particles parallel to the direction of shear, which results in increased face-to-face interaction of the particles (Skempton 1985). The resulting shear strength is low because it is difficult for the face-to-face particles to establish contact or bonding between them (Terzaghi et al. 1996). The establishment of a residual strength condition also results in increased water content at or near the preexisting failure surface (Skempton 1985). In summary, the particle contact and bonding that leads to a cohesion strength parameter greater than zero have been significantly reduced or removed by the shear displacement required to reach a residual strength condition. This results in only a frictional shear resistance that is represented by a residual friction angle and the effective normal stress acting on the shear surface. Because the residual strength is controlled by the frictional resistance of face-to-face particles, the residual strength is a function of clay mineralogy. As a result, it is recommended that a stress dependent strength envelope be used to model the drained residual strength and the resulting strength envelope should pass through the origin, i.e., the value of effective stress cohesion strength parameter should be zero, in stability analyses involving a residual strength condition. Alternatively, the stress dependent can be used directly in the stability analysis instead of a friction angle and cohesion.

EMPIRICAL CORRELATION FOR DRAINED FULLY SOFTENED SECANT FRICTION ANGLE FOR FINE-GRAINED SOILS

The history of comparing peak effective stress friction angle (ϕ') of normally clay soils with plasticity index (PI) can be traced back to the late 1950's. For example, Bjerrum and Simons (1960) present a relationship that relates ϕ' to plasticity index (PI) for normally consolidated soils. Although empirical correlations between ϕ' and PI are presented by Kenney (1959), Holt (1962), Brooker and Ireland (1965), Mitchell (1965), Bjerrum (1967) and Deere (1967), these correlations have considerable scatter which is noted by Kanji (1974). Subsequently, Skempton (1970) equated the fully softened shear strength of a soil, which corresponds to the random arrangement of clay particles, to the peak strength of a normally consolidated soil.

Skempton (1970) concludes that full softening reduces clay strength to the "critical state" strength so there is no further strength loss due to increase in water content and void ratio. However, additional strength loss can occur due to shear displacement. Field observations indicate that the fully softened strength can be mobilized around excavations in fissured clays (Skempton, 1977) and in desiccated, cracked, and weathered compacted clay embankments, e.g., levees, where infiltration of water along cracks results in higher water contents and void ratios (Wright et al., 2007). Softening around excavations is primarily due to stress relief that results in opening of fissures and development of negative pore-water pressures (Terzaghi, 1936). Softening in highway (Wright et al., 2007) and levee slopes is primarily due to cycles of desiccation and weathering that allows infiltration that results in clay swelling, an increase in moisture content, and a strength reduction.

Updated Empirical Correlation for Drained Fully Softened Secant Friction Angle

The empirical correlation for drained fully softened secant friction angle proposed by Stark and Eid (1997) and revised by Stark et al. (2005) only requires LL and CF to estimate the drained fully softened friction angle (ϕ'_{fs}). Thus, the fully softened strength correlation suggested by Stark et al. (2005) provides a reliable estimates of ϕ'_{fs} for use in preliminary design, verification of laboratory test results, and confirmation of back-analysis of first time slides.

The fully softened strength empirical correlation uses three different CF groups, i.e., CF $\leq$ 20%, 25% $\leq$ CF $\leq$ 45%, and CF $\geq$ 50%, which is similar to the residual strength correlation and accounts for the effect of CF and σ'_n on ϕ'_{fs} values. Furthermore, the empirical correlation uses values of LL and CF measured using disaggregated samples to make it similar to the empirical correlation for drained residual secant friction angle shown in Fig. 2.

The present study suggests a separate mathematical expression for each trend line of the correlation in Fig. 4 that can be used to estimate values of ϕ'_{fs} and a stress dependent strength envelope using values of LL and CF measured using disaggregated samples.

Equations for Updated Empirical Correlations for Drained Fully Softened Secant Friction Angle

Stark and Eid (1997) and Stark et al. (2005) present a relationship between LL and drained fully softened secant friction angle in graphical form with separate trend lines for each effective normal stress for three different CF groups. The present study considered each CF group separately while developing an equation for each trend line for the three effective normal stresses considered, i.e., 50, 100, and 400 kPa. The empirical correlation for fully softened secant friction angle in Stark et al. (2005) and Fig. 4 already includes an effective normal stress of 50 kPa so this trend line did not have to be added during this study but was an impetus for adding this effective normal stress to the residual strength correlation. Stark et al. (2005) adjusted the ring shear fully softened strength by adding 2.5 degrees to the measured values to make these comparable to the values obtained using a triaxial compression test and more importantly first-time landslides (Skempton, 1970). This adjustment was deemed necessary by Stark and Eid (1997) and Stark et al. (2005) because first-time landslides usually do not involve a horizontal failure surface as is present in the ring shear device. The failure surface in first-time slides is closer to the orientation of the failure surface in a triaxial compression test so the existing and new values of ϕ'_{fs} were increased by 2.5 degrees to reflect the triaxial mode of shear. New data for three natural soils tested herein has been added to the existing database with this adjustment of 2.5 degrees and the updated correlation is shown in Fig. 4.

A set of three equations was developed during the present study for the empirical correlation for drained fully softened secant friction angles of CF Group #1 and for LL values ranging from 30% to less than 80% (30% $\leq$ LL < 80%). These equations are given below as Equations (5.1) to (5.3). The LL range of 30 to 80% is specified because the ring shear data are available only for this LL range. A second degree polynomial can be used to represent the trend lines for CF Group #1 and for all three effective normal stresses.

$$\left(\phi_{r}\right)_{\sigma_{n}^{'}=50kPa} = 34.85 - 0.0709(LL) + 2.35x10^{-4}(LL)^{2} \tag{5.1}$$

$$\left(\phi_{r}\right)_{\sigma_{n}^{'}=100kPa} = 34.39 - 0.0863(LL) + 2.66x10^{-4}(LL)^{2} \tag{5.2}$$

$$\left(\phi_{r}\right)_{\sigma_{n}^{'}=400kPa} = 34.76 - 0.13(LL) + 4.71x10^{-4}(LL)^{2} \tag{5.3}$$

A set of three equations was also developed herein for CF Group #2 and LL values ranging from 30% to 130% (30% $\leq$ LL $\leq$ 130%) and is given below as Equations (6.1) to (6.3). A second degree polynomial was also used to represent the trend lines for CF Group #2 and for all three effective normal stresses.

$$\left(\phi_{r}\right)_{\sigma_{n}^{'}=50kPa} = 36.18 - 0.1143(LL) - 2.354x10^{-4}(LL)^{2} \tag{6.1}$$

$$\left(\phi_r\right)_{\sigma_n^{'}=100\mathrm{kPa}} = 33.11 - 0.107(\mathrm{LL}) + 2.2\mathrm{x}10^{-4}(\mathrm{LL})^2 \quad (6.2)$$

$$\left(\phi_r\right)_{\sigma_n^{'}=400\mathrm{kPa}} = 30.7 - 0.1263(\mathrm{LL}) + 3.442\mathrm{x}10^{-4}(\mathrm{LL})^2 \quad (6.3)$$

Another set of three equations was developed for CF Group #3 and LL values ranging from 30% to 300% (30% $\leq$ LL $\leq$ 300%) and is given below as Equations (7.1) to (7.3). A third degree polynomial also can be used to represent the trend lines for CF Group #3 and for all three effective normal stresses. The ring shear data along with the trend lines sketched by Stark et al. (2005) for CF Group #3, and the trend lines sketched from the newly developed Equations (7.1) to (7.3) are plotted on Fig. 5. Fig. 5 shows that the trend lines plotted using Equations (7.1) to (7.3) are in agreement with the trend lines suggested by Stark et al. (2005).

$$\left(\phi_r\right)_{\sigma_n^{'}=50\mathrm{kPa}} = 33.37 - 0.11(\mathrm{LL}) + 2.344\mathrm{x}10^{-4}(\mathrm{LL})^2 - 2.96\mathrm{x}10^{-7}(\mathrm{LL})^3 \quad (7.1)$$

$$\left(\phi_r\right)_{\sigma_n^{'}=100\mathrm{kPa}} = 31.17 - 0.142(\mathrm{LL}) + 4.678\mathrm{x}10^{-4}(\mathrm{LL})^2 - 6.762\mathrm{x}10^{-7}(\mathrm{LL})^3 \quad (7.2)$$

$$\left(\phi_r\right)_{\sigma_n^{'}=400\mathrm{kPa}} = 28.0 - 0.1533(\mathrm{LL}) + 5.64\mathrm{x}10^{-4}(\mathrm{LL})^2 - 8.414\mathrm{x}10^{-7}(\mathrm{LL})^3 \quad (7.3)$$

Drained Fully Softened Strength Envelope

Determining whether the value of effective stress cohesion should be equal to zero is a little more complicated for the fully softened condition than the residual strength condition. Skempton (1977) concludes that overconsolidated clays undergo a softening process that results in the fully softened strength being mobilized, not the shear strength of the intact or unsoftened overconsolidated clay, in slopes that have not undergone previous sliding (first-time slides). This softening process reduces the effective stress cohesion component of the Mohr- Coulomb shear strength parameters but does not cause orientation of clay particles or a reduction in the friction angle (Skempton 1970). Because Skempton (1970) concludes that softening over time reduces clay strength to the “critical state” strength, where further distortion or weathering will not result in any change in water content, the strength is approximately equal to the strength of the soil when it is normally consolidated. For London clay, the difference between the "critical state" strength (22.5 degrees) and the peak strength of normally consolidated London Clay (20 degrees) is about 2.5 degrees (Skempton, 1970). Thus, Skempton (1970) concludes that equating the strength of normally consolidated test specimens to the fully softened strength is a "somewhat conservative" approximation.

Because the fully softened shear strength corresponds to the drained peak strength of a normally consolidated specimen, this suggests that the value of effective stress cohesion (c') should be set to zero, i.e., the value of cohesion measured in shear tests on normally consolidated clay (Holtz and Kovacs 1981, Terzaghi et al. 1996), for the analysis of first time slides in overconsolidated clays. This is important because even small values of c' can result in significant differences in calculated factors of safety especially in shallow slides, such as levee or embankment slopes. However, back analysis of first-time slides in London clay, indicate small values of c', approximately 0.96 kPa (20 psf), can be mobilized (Chandler and Skempton, 1974). Skempton (1977) also suggests a c' of 0.96 kPa (20 psf) and ϕ'_{fs} of 20 degrees for London clay. Mesri and Abdel-Ghaffar (1993) back-analyzed forty-five case histories and conclude c' can range from zero to 24 kPa. In summary, the fully softened value of cohesion should be zero unless back-analysis of local case histories suggests a value greater than zero.

SPREADSHEET FOR EMPIRICAL CORRELATIONS

During the present study a spreadsheet was developed that utilizes only two parameters, CF and LL, as input and generates values of ϕ'_r and ϕ'_{fs} for effective normal stresses of 0, 50, 100, 400, and 700 kPa. The spreadsheet uses the equations for the various trend lines presented herein and is an electronic supplement to this paper. The estimated stress dependent residual and fully softened strength envelopes are plotted on a single figure in the spreadsheet as well as tables of shear stress and effective normal stress for the fully softened and residual strength envelopes. The tabulated values of effective normal stress and shear stress can be used directly in slope stability software to describe the stress dependent strength envelope instead of using values of effective stress cohesion (c') and friction angle (ϕ'). The advantage of showing both drained residual and fully softened strength envelopes on the same graph is the user can compare the difference between the fully softened and residual strengths in a single figure to determine the importance of identifying whether or not a preexisting shear surface is present or will develop in the slope and if so, how much of the failure surface should be assigned a residual strength.

EFFECT OF SAMPLE PREPARATION ON STRENGTH AND INDEX PROPERTIES

Effect of Sample Preparation on ϕ'_r

The drained residual strength is a fundamental property because the soil structure, stress history, particle interference, and diagenetic bonding have been removed by continuous shear displacement in one direction (Stark et al., 2005). As a result, the residual strength is controlled by the frictional resistance of individual clay mineral particles, oriented primarily face-to-face, sliding across one another. The frictional shear resistance induced by sliding of individual clay mineral particles is controlled by the fundamental characteristics of the clay particles, e.g., type of clay mineral(s) and quantity or percentage of the clay mineral(s). Thus, laboratory preparation and shear testing must be able to disaggregate the clay mineral particles so they can be individually oriented parallel to the direction of shear.

Skempton (1964) suggests field shear movement of about a meter is required to achieve a residual strength condition. This field shearing causes an increase in fines content of the material along the shear surface by pushing silt and sand sized particles away from the shear surface. Mesri and Cepeda-Diaz (1986) conclude “the shearing process itself disaggregates and orients even the clay plates at the surface of aggregates adjacent to the shear plane.” Chandler (1969) measured a higher CF in the shear surface than the overall specimen also indicating disaggregation during shear. Thus during the process of achieving a residual strength condition, aggregated clay mineral particles are disaggregated close to individual clay mineral particle size, which must be duplicated in laboratory testing to achieve a residual strength that is consistent with field conditions.

A remolded soil sample is preferred to an undisturbed shear surface specimen for laboratory residual shear strength testing because of difficulties in sampling, orienting, and shearing in the direction in which shearing had occurred in the field. However, the use of a remolded specimen results in a larger shear displacement being required in the laboratory to disaggregate the clay mineral particles and achieve a residual strength condition than an undisturbed specimen from the shear surface. The laboratory displacement required to reach a residual strength condition can be reduced by using a disaggregated sample and preshearing the resulting test specimen prior to drained shearing. For highly indurated soils, the disaggregation of clay mineral particles in the laboratory can be facilitated by ball milling or pulverizing by some other means, such as disc milling, rod milling, and/or blending, to process the soil through Number 200 sieve to simulate field disaggregation. Silt and clay sized particles that show no induration/aggregation do not require ball milling because ball milling may change the texture and gradation of such soils.

In summary, disaggregation of highly indurated materials was used in this study to facilitate measurement of the drained residual strength using remolded samples of overconsolidated clays, mudstones, claystones, and shales in the laboratory and to simulate field disaggregation processes that occurs over many years and large shear displacement. Soils with little or no induration/aggregation were only pulverized using a mortar and pestle after air drying and processed through the Number 40 sieve for ring shear and index property testing. Because values of LL and CF measured on disaggregated samples are in better agreement with true values of ϕ'_r, values of LL and CF measured using ball milled samples were used to develop the empirical correlation in Fig. 2 so values of LL and CF measured using disaggregated samples should be used to estimate values of ϕ'_r from Fig. 2.

Effect of Sample Preparation on ϕ'_{fs}

The use of values of LL and CF measured using disaggregated samples of overconsolidated clays, mudstones, claystones, and shales to develop the ϕ'_{fs} correlation in Fig. 4 is less intuitive than described above for the ϕ'_r correlation. In theory, the fully softened shear strength represents the strength of a soil when the effects of overconsolidation are removed. Thus the strength corresponds to a normally consolidated soil and it reflects the ability of particles to establish short range and random interaction and interlocking (Mesri and Cepeda-Diaz, 1986). The fully softened shear strength is measured at a small shear displacement so significant reorientation of the particles parallel to the direction of shear has not occurred. Thus, particle size and shape do affect the measured value which should be greater than the frictional shear resistance of reoriented particles. It is expected that during mobilization of the fully softened shear strength in the field, the clay particles retain at least some of their natural structure and there is little orientation of clay particles along the shear surface in the direction of shear as happens in achieving a residual strength condition.

In the field, the clay particles may be close to or completely disaggregated at the fully softened strength condition in highly weathered clays, silts, and compacted clayey fills. If it is desired to simulate full or complete weathering for evaluation of highly weathered clays, silts, and compacted clayey fills, the air-dried soil should be processed through the U.S. Standard Sieve Number 40 and further disaggregated using a blender after soil hydration for at least 48 hours. The fully softened strength and index property testing of this material should be performed using the blenderized soil.

Clay mineral particles may still have some aggregation in indurated, e.g., e.g., claystones, shales, mudstones, materials and in highly plastic, highly overconsolidated clays at the time of sampling even though it is believed a fully softened condition exists. To simulate this level of disaggregation, inudurated materials should be processed through the Number 40 sieve and then further disaggregated using a blender after soil hydration for at least 48 hours to reflect a "fully" weathered condition before strength testing as suggested above. However, there is some field evidence that even indurated materials, e.g., claystones, shales, mudstones, may over the life of a project become so weathered that their particles are substantially disaggregated but not reoriented parallel to the direction of shear, i.e., residual strength condition. If full disaggregation of indurated material due to weathering and softening is anticipated or desired for design or analysis purposes, the material could be ball or disc-milled, processed through the

Number 200 sieve, hydrated, and then blenderized before fully softened strength and index property testing to reflect a “fully” weathered condition.

In all preparation procedures, care must be exercised for those materials containing non-clay or silt sized particles to avoid breakdown of these larger particles during processing and changing the gradation of the soil. Whatever sample preparation procedure is used to simulate a “fully” weathered condition, it should be carefully documented so the resulting data can be properly interpreted for design and properly compared to existing fully softened strength correlations such as the one presented herein.

Empirical correlation for drained fully softened secant friction angles in Fig. 4 was developed using LL and CF values measured using disaggregated samples to make it compatible with the empirical correlation for drained residual secant friction angles and easy comparison by the users. To facilitate use of the empirical correlations, adjustment factors to estimate values of LL and CF measured using disaggregated samples from ASTM derived values of LL and CF are discussed below.

Effect of Sample Preparation on LL and CF

Because the empirical correlations for drained residual and fully softened secant friction angle use index properties measured using disaggregated soil samples for overconsolidated clays, mudstones, claystones, and shales, the effect of sample preparation on index properties is discussed in this section. Non-aggregated materials, e.g., silty clay or clayey, compacted fill, were not ball milled and only processed through Number 40 sieve as required by ASTM D4318 and D422 (ASTM, 2010c and 2010d) for measuring LL and CF, respectively.

Preparation of a remolded specimen can influence the measured LL. For example, La Gatta (1970) shows that disc milling Cucaracha shale from the Panama Canal for six minutes resulted in an increase in LL from 49% to 156%. La Gatta (1970) and Townsend and Banks (1974) suggest that the degree of induration (aggregation) that survives a particular sample preparation procedure will influence the measurement of the index properties. Mesri and Cepeda (1986) conclude that most heavily overconsolidated clays, mudstones, claystones, and shales possess varying degrees of induration. This induration involves diagenetic bonding between clay mineral particles by carbonates, silica, alumina, iron oxides, and other ionic complexes. Mesri and Cepeda (1986) suggest ball milling of highly overconsolidated clay specimens to “free” or disaggregate the clay mineral particles. Ball milling is suggested and only used herein for highly overconsolidated clays, mudstones, claystones, and shales because they possess substantial diagenetic bonding that is usually not destroyed using a mortar and pestle. To measure LL, which is used herein to infer clay mineralogy, the indurated mudstone, claystone, and shale samples are air-dried, disaggregated by ball-milling, and processed through a Number 200 sieve. Disaggregation of highly overconsolidated particles by ball/disc milling, blending/grinding (Townsend and Banks, 1974), or other suitable means, results in a better estimate of the actual LL than ASTM test methods (2010a) because more of the diagenetic bonding and induration is eliminated which allows more particle surface area to be exposed and hydrate than if the clay particles are not disaggregated. Thus, a clay sample with disaggregated particles usually results in a higher LL than that obtained using the ASTM (2010a) test method.

Normally to lightly overconsolidated fine-grained materials that are not indurated (aggregated), do not require disaggregation of clay mineral particles. As a result, these materials should be processed through Number 40 sieve as suggested by ASTM (2010a) and used for measuring LL and CF. The ball/disc milling of non-indurated clay materials could result in changing the texture and gradation of the soil. Therefore, judgment is required to determine whether or not a material should be ball-milled or not (Stark et al., 2005). This decision can be made after examination of the material and determining whether the materials can be sufficiently broken down with a mortar and pestle to disaggregate the clay particles. If not, ball/disc milling or any other means (blending/grinding) should be used to disaggregate the clay particles so the material can be processed through Number 200 sieve.

Stark et al. (2005) present a relationship between ball milled derived LL and ASTM derived LL using fourteen soil samples of highly overconsolidated clays. The correlation suggested by Stark et al. (2005) facilitates the estimation of ball milled derived LL values from the ASTM derived LL because ball/disc milling requires special equipment and extra effort that may not be readily available in practice. Because commercial laboratories primarily, if not exclusively, utilize the ASTM (2010a) to measure LL, this correlation can be used to estimate the ball milled derived LL to obtain a representative value of LL for highly indurated materials.

Test results for fifteen additional soils, i.e., a total 29 soils, were used herein to develop a new relationship between the ASTM derived LL using the Number 40 sieve, referred herein as $LL_{\#40}$, and the LL measured on a sample processed through Number 200 sieve, referred herein as $LL_{\#200}$. The resulting relationship is expressed in Equation (8) and can be used with an ASTM derived value of LL ($LL_{\#40}$) to estimate the $LL_{\#200}$ value for a particular soil. This should reduce the need for commercial laboratories to ball/disc mill overconsolidated clays, claystones, shales, and mudstones and process the material through a Number 200 sieve to estimate representative values of LL and strength envelopes.

$$\frac{LL_{\#200}}{LL_{\#40}} = 0.003\left(LL_{\#40}\right) + 1.23 \tag{8}$$

Equation (8) shows the ratio of $LL_{\#200}/LL_{\#40}$ increases with increasing $LL_{\#40}$. It is anticipated that the higher the LL, the greater the bonding between clay mineral particles and the more difficult disaggregation of the clay particles becomes, which results in the greatest difference between values of $LL_{\#40}$ and $LL_{\#200}$. Thus, high plasticity claystones, shales, and mudstones should be processed through Number 200 sieve before measuring LL or the LL values adjusted using Equation (8).

The field conditions under which the residual, and possibly fully softened, strength are mobilized, i.e., disaggregated clay particles, also results in a higher CF. In other words, along a preexisting shear surface the soil particles will be disaggregated so the LL and CF should reflect this field condition. ASTM (2010b) derived CF, called herein $CF_{\#40}$, and CF measured using material processed through Number 200 sieve, called herein $CF_{\#200}$, were used to develop a correlation between these values of CF. Because commercial laboratories primarily utilize ASTM (2010b) to measure CF, Stark et al. (2005) present a relationship between $CF_{\#40}$ and $CF_{\#200}$ measured using samples of fourteen different highly overconsolidated clays. The present study used test results for eighteen additional soils, i.e., a total of 32 soils, to develop the relationship in Equation (9). Equation (9) can be used to estimate the value of $CF_{\#200}$ which may be a more representative of the field value of CF for highly indurated clays.

Equation (9) shows the $CF_{\#200}/CF_{\#40}$ ratio decreases as $CF_{\#40}$ increases. It is anticipated that this decrease is caused by the $CF_{\#40}$ value being in better agreement with the $CF_{\#200}$ value at higher values of CF. Stark et al. (2005) suggest that this may be attributed to the dispersing agent, sodium hexametaphosphate, being more effective in high plasticity soils than low plasticity soils or processing the material through the Number 40 sieve is sufficient to disaggregate the material because CF is so high.

$$\frac{CF_{\#200}}{CF_{\#40}} = 0.0002\left(CF_{\#40}\right)^2 - 0.0278\left(CF_{\#40}\right) + 2.15 \qquad (9)$$

SUMMARY AND CONCLUSIONS

Updated empirical correlations for drained residual and fully softened secant friction angles using LL, CF, and effective normal stress are presented. For consistency between the empirical correlations for drained residual and fully softened secant friction angles, ϕ'_r and ϕ'_{fs} are plotted against LL and CF measured using soil processed through the Number 200 or coarser sieve for highly indurated clays and shales and processed through the Number 40 sieve for all other soils.

The trend lines in the new empirical correlations for drained residual and fully softened secant friction angles are modeled using mathematical expressions to facilitate distribution and use in practice. A separate equation is developed for each trend line in each CF group. The mathematical expressions reduce the need to utilize the graphical version of the empirical correlations.

A spreadsheet developed during the present study incorporates the mathematical expression for each trend line and is submitted as an electronic supplement to this paper. The spreadsheet requires only two input parameters, LL and CF, and generates values of ϕ'_r and ϕ'_{fs} for different effective normal stresses that can be used to develop the stress dependent failure envelopes. The resulting stress dependent residual and fully softened strength envelopes can be used directly in slope stability analyses for preexisting and first-time landslides, respectively, and are compared to assess the importance of identifying the presence or development of a shear surface.

Sample preparation affects LL and CF values therefore these parameters should be measured using the following procedure:

- Clay soils with little or no induration (aggregation) should by processed through Number 40 sieve in accordance with relevant ASTM test methods (2010 a and b) for both residual and fully softened strength testing.

- Highly indurated (aggregated)/heavily overconsolidated clays and shales should be disaggregated by processing the material through the Number 200 sieve using ball milling or some other disaggregation method for residual strength testing and possibly for fully softened strength testing depending on the degree of weathering and softening to be simulated.

ASTM derived values of LL and CF measured using soil samples processed through Number 40 sieve can be converted to LL and CF values measured using disaggregated soil samples by empirical expressions in Equations (8) and (9) to estimate stress dependent strength envelopes from the correlations presented herein.

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FIGURE CAPTIONS:

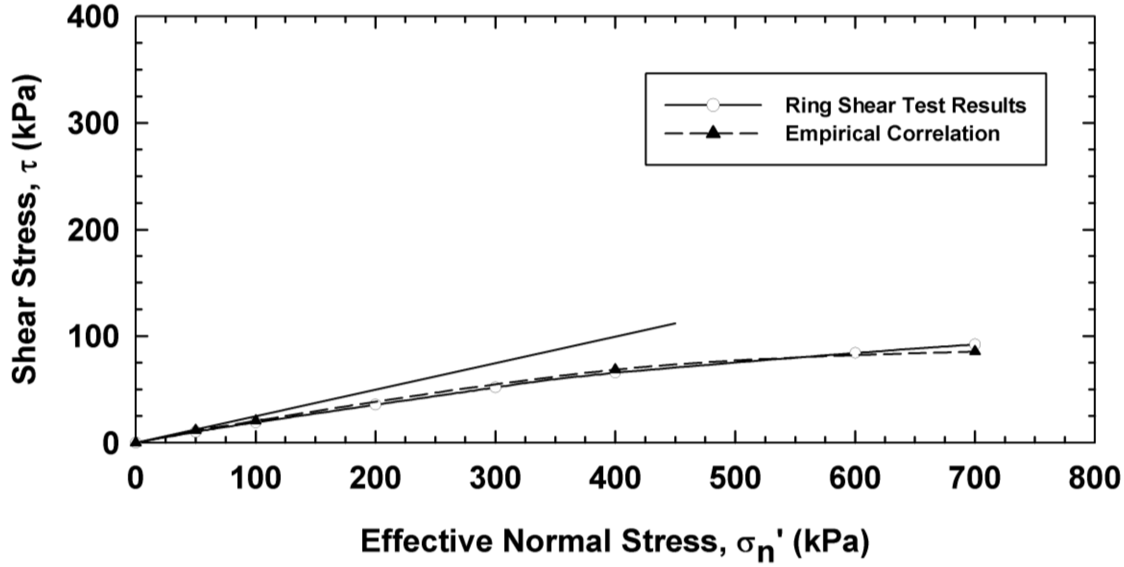
Fig. 1. Stress dependent drained residual strength envelope from ring shear test results and empirical correlation from Stark et al. (2005)

Fig. 2. Updated empirical correlation for drained residual secant friction angle based on liquid limit (LL), clay-size fraction (CF), and effective normal stress (σ'_n) for seventy-three natural soils

Fig. 3. Comparison of drained residual secant friction angle trend line suggested by Stark et al. (2005) and those estimated using Equations (1.1) to (1.4) for CF Group #1 (CF<20%)

Fig. 4. Updated empirical correlation for drained fully softened secant friction angle based on liquid limit (LL), clay-size fraction (CF), and effective normal stress (σ'_n) for thirty-nine natural soils

Fig. 5. Comparison of drained fully softened secant friction angle trend lines suggested by Stark et al. (2005) and those estimated using mathematical Equations (7.1) to (7.3) for CF Group #3 (25%≤CF≤45%).



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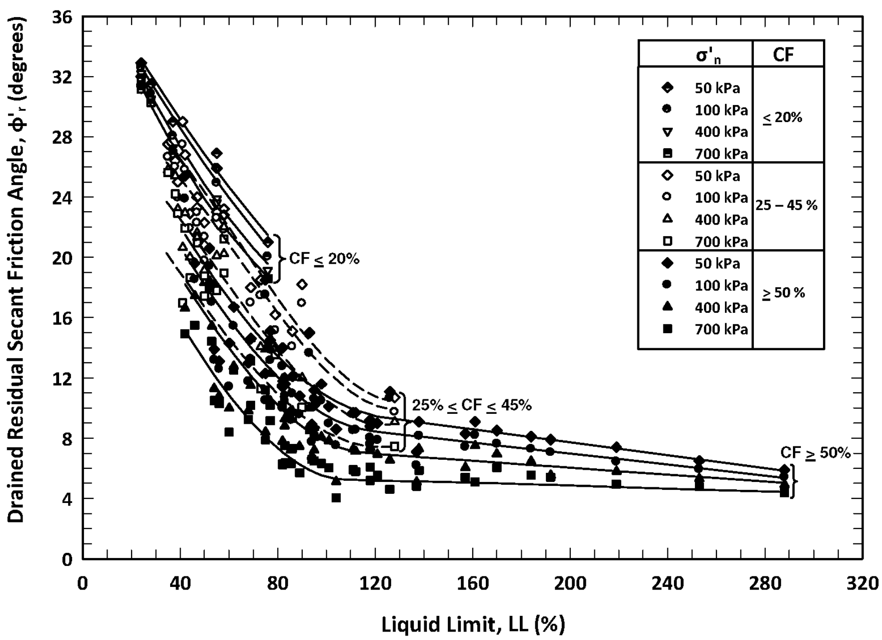

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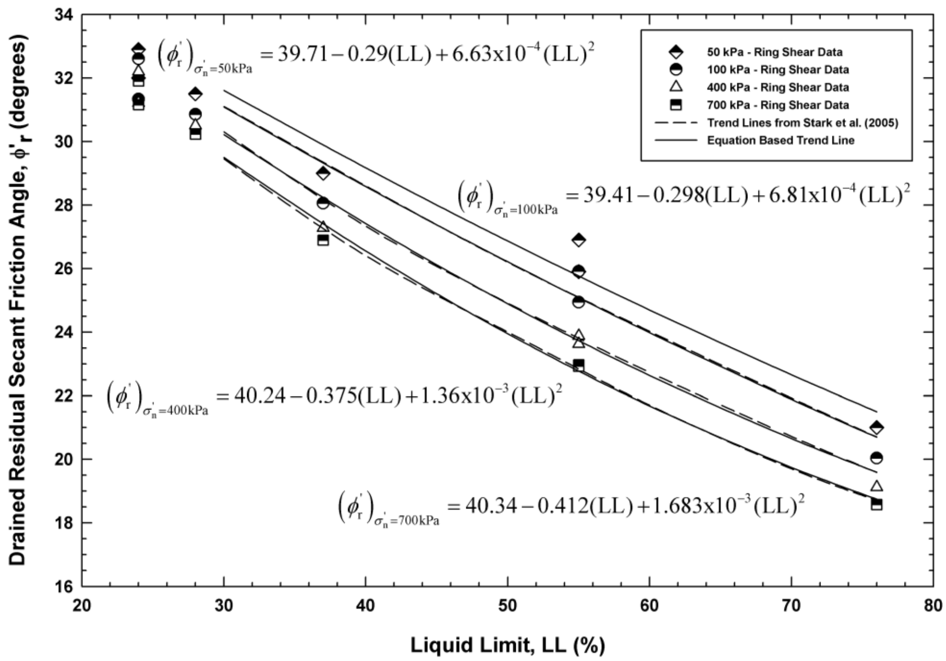

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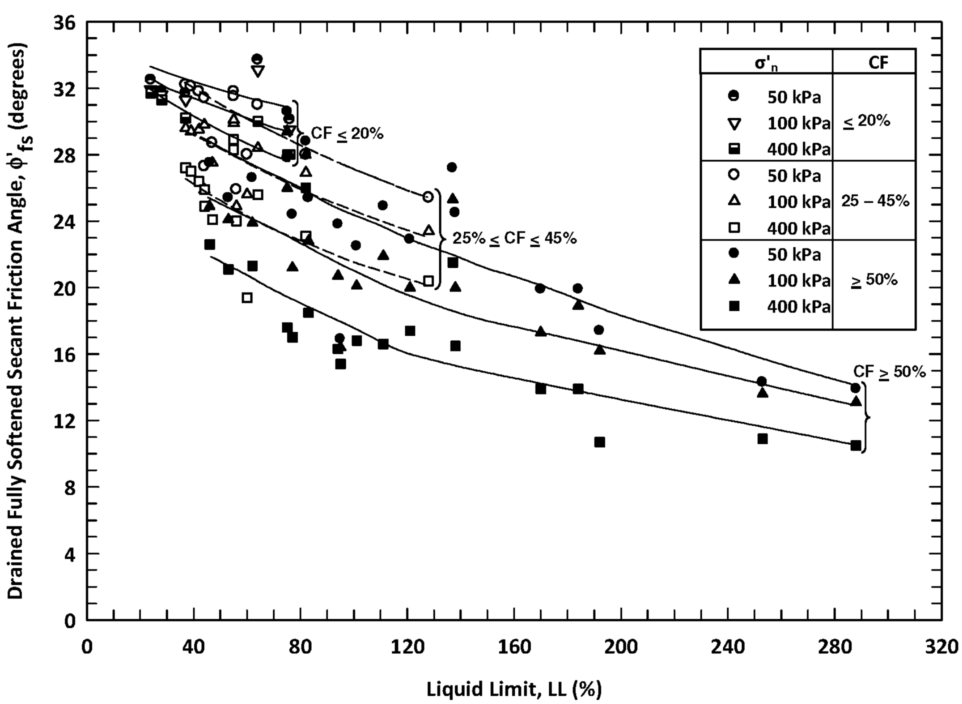

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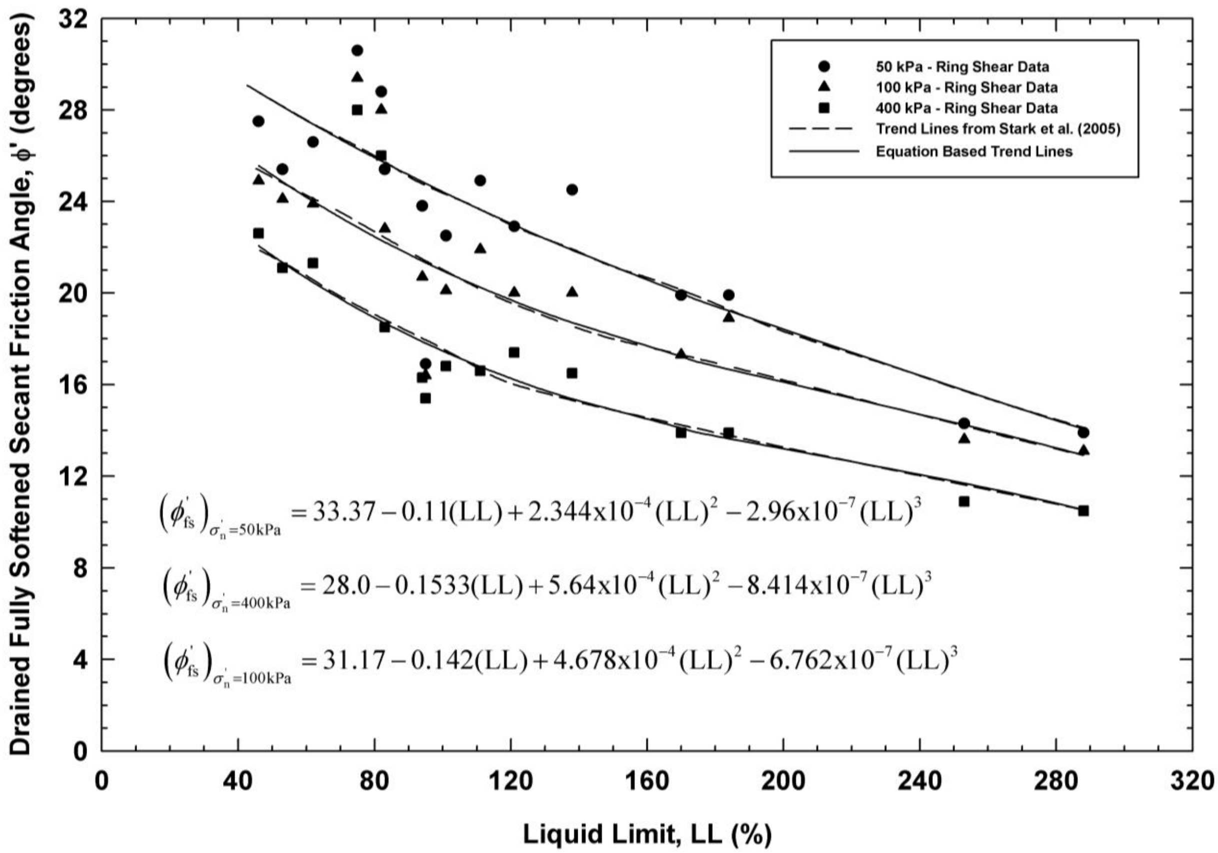

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