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THE IMPORTANCE OF A DESICCATED CRUST ON CLAY SETTLEMENTS

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ABSTRACT

Analysis of the settlement at five sites on San Francisco Bay Mud and two sites on Boston Blue Clay has shown that where dessicated crust has developed on the clay, spatial variations in the preconsolidation pressure are the most important cause of differential settlements. A simple probabilistic method is described that can be used to establish confidence limits for settlements at sites with a desiccated crust. Variations in preconsolidation pressure are accounted for by estimating confidence limits for a parameter termed the "radius of preconsolidation.'' Application of the methodology to the settlement of fills on San Francisco Bay Mud and Boston Blue Clay has shown that the method produces results in good agreement with field measurements and that it is suitable for use in practice. The method can be used for clay deposits which have an overconsolidated layer and requires no more testing than is performed for conventional settlement analyses. However, unlike conventional settlement analyses, it provides an estimate of the likelihood that the settlements will vary from the mean by a given amount due to spatial variations in soil properties.

Key words: consolidation, settlement, soil mechanics, statistical analysis (IGC: E2)

INTRODUCTION

Conventional methods of estimating settlements due to consolidation of clay are deterministic. These methods, described in most soil mechanics and shallow foundation textbooks (e.g. Leonards, 1962; Terzaghi and Peck 1967; Lambe and Whitman, 1969; Sowers, 1979; Holtz and Kovacs, 1981; U.S. Navy, 1987), make use of laboratory tests to measure compressibilities and preconsolidation pressures, and elastic theory to estimate stresses due to fills and buildings. In most cases calculations are made using average values of compressibilities and preconsolidation pressures. and best estimates of stresses. Sometimes, to estimate extreme possible settlements, upper limit values of compressibility and lower limit

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values of preconsolidation pressures are used in the calculations.

The experience with these conventional methods has generally been good. When relatively undisturbed samples are available, the magnitudes of the settlements calculated using average soil properties are usually in reasonable agreement with observed settlements, a fact responsible for the continued use of conventional procedures in engineering practice.

Differential settlements can be easily estimated using the conventional method when they are due to (a) differences in thickness of compressible soils, or (b) differences in stresses at different locations, for example, beneath the center and the corner of a building. Given uniform thickness and the same induced stresses, however, the conventional method predicts uniform settlement. Field observations show, however, that differential settlements occur in clays with a desiccated crust even where there is no apparent systematic difference in clay thickness or induced stresses. Differential settlements occur due to random variations of compressibility and preconsolidation pressure within the clay and crust.

BAY FARM ISLAND–AN EXAMPLE OF NONUNIFORM SETTLEMENTS DUE TO RANDOM VARIATIONS IN SITE CONDITIONS

An example of the occurrence of differential settlements under conditions of similar clay thickness and similar induced stress is afforded by the settlements observed at Bay Farm Island on the east side of San Francisco Bay in California. As shown in Fig. 1, Bay Farm Island is located south of the island of Alameda, and north of Oakland International Airport.

Bay Farm Island consists of two separate areas. The eastern part of the site, called the "crusted area", was drained beginning in the 1920s. The crust blanketing this part of the site developed during a period of approximately 40 years when the area was used for farming. The western part of the site was not drained prior to its development in the 1960s, and thus there was little or no dried crust on this part of the site when the area was filled.

A cross-section through the site prior to hydraulic filling is shown in Fig. 2. The results

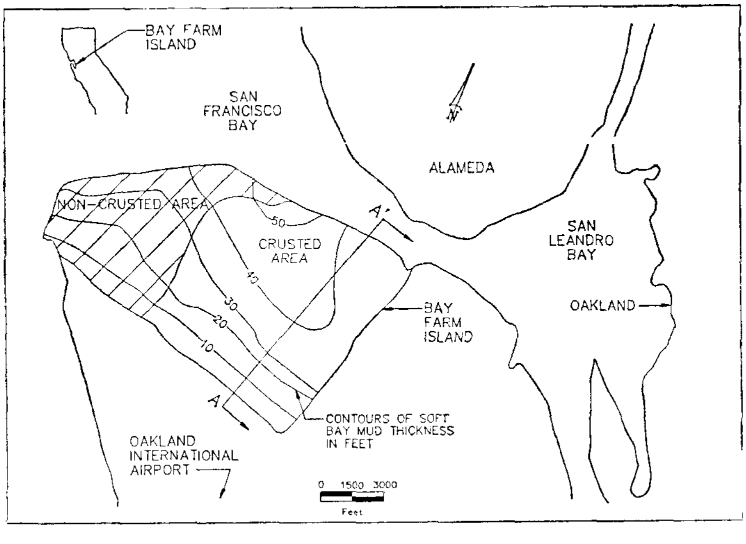


Fig. 1. Site plan and contours of Bay Mud thickness at Bay Farm Island

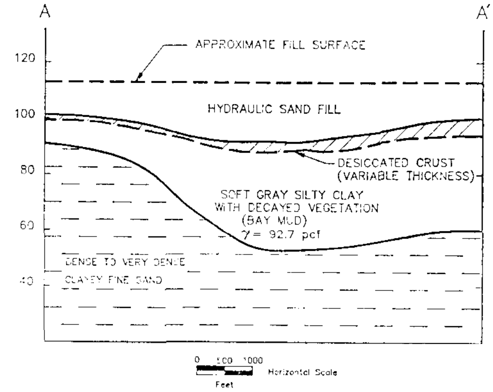


Fig. **2.** Cross-section **A-A'** at Bay Farm Island

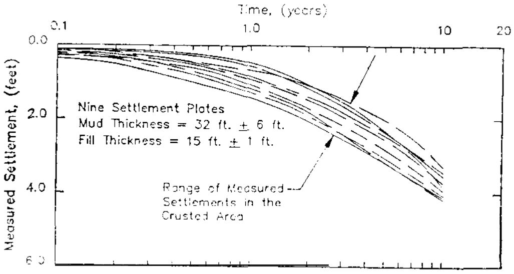


Fig. **3.** **Measured** settlements for the crusted area at Eay Farm Island

of 32 borings, 10 in the non–crusted ares and 22 in the crusted area, were used to determine the subsurface conditions. The spacing of the borings ranged from 100 feet (30 m) to 1000 feet (300 m) with an average spacing of 500 feet (150 m) in the crusted area and 680 feet (210 m) in the non–crusted area. The site is underlain by San Francisco Bay Mud, a plastic gray silty clay. At the Bay Farm Island site the natural water content of the Bay Mud prior to filling varied from 58% to 110%, with an average of 83%. The Liquid Limit varies from 77 to 95 with an average of 85, and the Plastic Limit varies from 36 to 47 with an average of 40. The thickness of the Bay Mud varies from 10 ft. (3.1 m) to 55 ft. (16.8 m), increasing toward the northwest. Beneath the Bay Mud is a layer of dense to very dense clayey fine sand. Field observations of piezometric levels showed that this layer provided drainage at the base of the Bay Mud.

The entire site—both the crusted and the non–crusted areas—was filled to approximately elevation 113 feet (34 m) during 1966 and 1967 with 8 ft. (2.4 m) to 20 ft. (6.1 m) of hydraulic sand fill. This fiill was left in place approximately 12 years prior to final grading and commercial development of the area. Forty–seven settlement plates were installed to monitor the settlements due to the weight of the fill, and thirty two observation wells were used to monitor water levels. About ten years of observations provide a very useful body of data concerning the settlements that occurred at the site.

The settlement observations at Bay Farm Island showed one thing very clearly, even at locations where the thickness of mud and the thickness of fill were the same, the settlements were not. Examples of measured variations in settlements for similar conditions of mud thickness and fill thickness are shown in Figs. 3 and 4.

The data shown in Fig. 3 are for nine settlement plates in the crusted area. The conditions of mud thickness and fill thickness at the settlement plates were similar but not equal, as indicated by the note in the Figure. One year after fill placement the settlements varied from about 0.4 ft. (0.1 m) to about 1.4 ft. (0.4 m). Ten years after fill placement the settlements varied from 3.2 ft. (1.0 m) to 4.3 ft. (1.3 m). Careful examination of the data showed that the settlements of the settlement plates in the crusted did not vary systematically with fill thickness or mud thickness. Thus, in this particular group of settlement plates, which are considered together because they all have approximately equal mud thickness and fill thickness, there is a significant amount of variation in settlement that cannot be attributed to systematic effects. The scatter appears to be due to the natural variation in the magnitude and depth of the desiccated crust.

The data shown in Fig. 4 are for seven settlement plates in the non–crusted area. As shown by the note in the figure, the mud and fill thicknesses were similar for all seven settlement plates. After one year, the measured

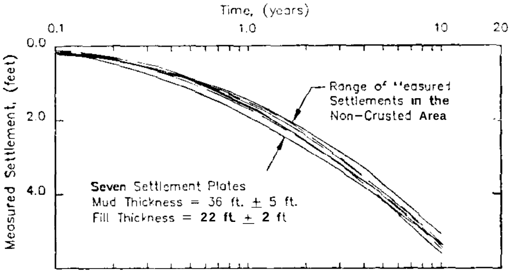
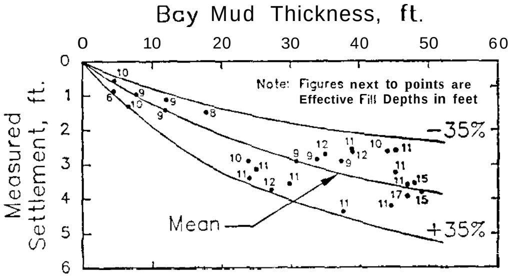


Fig. 4. Measured settlements for the non-crusted area at Bay Farm Island

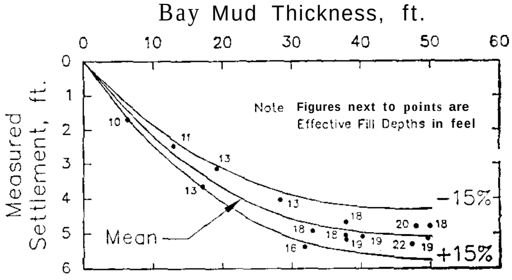
settlements ranged from 1.5 ft. (0.46 m) to 1.9 ft. (0.58 m). After ten years they ranged from 5.2 ft. (1.6 m) to 5.6 ft. (1.7 m). The settlements are larger than the crusted area but the variation is significantly smaller.

An alternative way of viewing the data from Bay Farm Island is shown in Fig. 5, where all settlement measurements eight years after fill placement are plotted against mud thickness. A mean line has been drawn through the points for the crusted area, in the upper part of Fig. 5, and another through the points for the non-crusted area, in the lower part of Fig. 5. In the crusted area, the maximum and minimum measured settlements vary ±35% from the mean and in the non-crusted area the variation is significantly less, about ±15% from the mean.

There appears to be no systematic correlation between settlement and mud or fill thickness for the settlements in the crusted area. For example, the "effective" fill thickness (described subsequently) ranged from 9 to 12 ft. (2.7 to 3.7 m) at a mud thickness of 40 ft. (12.2 m). An "effective" fill thickness of 12 ft. (3.7 m) resulted in only 2.7 ft. (0.8 m) of settlement while a fill thickness of 11 ft. (3.3m) caused 4.4 ft. (1.3 m) of settlement. This anomaly is due to the variability of desiccation in the crusted area. As expected, an increase in fill thickness usually resulted in an increase in settlement in the non-crusted area. In summary, Figs. 3 through 5 clearly illustrate that the measured settlements in the crusted area are smaller and more variable than the non-crusted area for similar mud and fill thicknesses.



(a) Crusted Area



(b) Non-Crusted

Fig. 5. Measured settlements in (a) Crusted and (b) Non-crusted area 8 years after fill placement, Bay Farm Island

Conventional methods, using average values of compressibility and preconsolidation pressure, would predict the average settlement quite accurately, as will be shown in a subsequent section of this paper. However, these conventional methods provide no indication of possible variations from the mean values of settlement. As shown in Fig. 5, the settlements at Bay Farm Island scattered as much as ±35% from the average in the crusted area for the same mud and fill thicknesses. The conventional method of estimating clay settlements provides no means for estimating the likelihood that the settlement will vary from the average by a given amount due to random variations in soil properties.

The procedure described in subsquent sections of this paper was developed to predict the magnitude and likelihood of deviations from the average settlements of clay. It follows the logic of the conventional method of analysis, and uses some simple concepts of probability to arrive at an estimated range of settlements for a given probability, rather than a single mean value.

PREVIOUS APPLICATIONS OF PROBABILITY TO CONSOLIDATION SETTLEMENTS

Folayan (1968) and Folayan et al. (1970) applied the theory of probability to calculation of settlements on San Francisco Bay Mud for the purpose of determining the optimum thickness of fill in a development. They showed how decision theory could be used to determine the optimum thickness of fill and the reliability with which the optimum can be determined.

Diaz–Padilla and Vanmarcke (1974) approached the uncertainty in soil properties by combining the soil properties into a pseudo–subgrade reaction factor and treating it as a random variable. The expected value and variance were estimated using a Taylor–series expansion about the mean values of the applied load and the subgrade reaction factor. Their analysis showed that clay settlements are very sensitive to uncertainties in preconsolidation pressure. The scatter in the settlements measured at Bay Farm Island support this conclusion. In particular, the variation in measured settlements is significantly larger in the crusted area, where the preconsolidated crust plays an important role in determining the magnitude of the settlement.

Vanmarcke and Fuleihan (1975) used a Taylor–series expansion to estimate the mean and variance of levee settlements. They concluded that variability in the compression ratio was the major contributor to settlement variability for a normally consolidated clay, and that variability in applied stress did not result in significant variations in settlement.

Freeze (1977) and Chang and Soong (1979) used probability density functions to model soil properties in a one–dimensional consolidation problem. Both studies concluded that spatial variations in soil properties can produce significant variations in consolidation settlements.

Ang and Tang (1984) utilize a performance function to estimate the probability of excessive settlement due to primary consolidation of a clay layer. The settlement parameters are varied until the performance function yields a particular failure condition, e. g., 2.5 inches (6.4 cm). The probability of failure is then calculated using the soil parameters at the failure point. Their model assumes that all the soil variables are normally distributed and neglects the contribution of secondary compression.

These studies all show that variations in settlements on clay can occur as a result in variations in soil properties and preconsolidation pressures. The purpose of this paper is to describe a simple probability procedure for incorporating considerations of variability in analyses of clay settlements, and to illustrate its use for calculating settlements on San Francisco Bay Mud and Boston Blue Clay.

VARIABILITY OF SOIL PROPERTIES

A considerable number of test results on samples taken prior to filling were available for the soils from Bay Farm Island, and these provide a basis for determining which properties contribute most to variations in settlement from place to place. Table 1 contains a statistical summary of these measured properties. Table 1 shows the number of measurements of each property, the average value, the standard deviation of the measured values, and the coefficient of variation (standard deviation divided by the average, expressed in percent).

The unit weight of the Bay Mud varies relatively little from place to place. The coefficient of variation is only 4.2 percent. Analysis of the measured values indicates that they may be approximated as normally distributed. Thus about two–thirds of the measured values would fall within one standard deviation (±4.2 percent) from the average, and about 95 percent of the measured values should fall within 1.96 standard deviations (t-8.4 percent) from the average. Due to the

Table 1. Statistical evaluation of soil properties at Bay Farm Island

Property	Sample Size	Mean	Standard Deviation	Coefficient of Variation (%)
Unit Weight of Bay Mud, (pcf)	18	92.7	3.9	4.2
Effective Fill Depth, (feet)				
1.) Crusted Area				
A.) 25 feet Bay Mud	4	10.6	0.39	3.7
B.) 45 feet Bay Mud	5	11.0	0.41	3.7
2.) Non-Crusted Area				
A.) 40 feet Bay Mud	3	18.5	0.15	0.8
B.) 48 feet Bay Mud	4	19.8	2.02	1.2
Compression Ratio, $C_{\varepsilon c}$	44	0.305	0.047	15.5
Recompression Ratio, $C_{\varepsilon r}$	43	0.029	0.009	31.0
Secondary Compression Ratio, C_α	14	0.016	0.005	32.0
Coefficient of Consolidation, C_v, (ft.^{2}/year)				
Taylor's Method				
1.) Virgin	54	16	15	49
2.) Recomprssion	23	211	168	80

Note: 1 pcf =0.1571 kN/m^3
1 foot=0.305 m

large number of borings at Bay Farm Island, the thickness of the Bay Mud was not considered to be a variable.

At Bay Farm Island, the water level was initially at or close to the natural ground surface and rose as the hydraulic fill was placed. Subsequently, the water table dropped at a rate slightly faster than that of the fill subsidence, although seasonal fluctuations resulted in short term variations in the pattern. As a consequence of the variations in water level, the load acting upon the mud varied somewhat. An "effective fill depth" was used in interpreting the measured settlements and in calculating settlements at various plate locations. This average fill load was determined by first averaging both fill thickness and depth to the water table for each year and then averaging these yearly values over the period of time for which readings were available. The depth of fill weighing 110 pounds per cubic foot (17.3 kN/m^3) which would produce the same loading was then determined, and was termed the "effective fill depth." It can be seen from Table 1 that the average effective fill depth varied little from one settlement plate location to another. The coefficient of variation for the effective fill thickness was only 3.7 percent for the crusted area, and approximately 1.0 percent for the non-crusted area.

Because the values of Bay Mud unit weight and effective fill thickness varied relatively little from place to place, these variables were considered to be constant in the settlement analysis described subsequently.

The Kolmogorov-Smirnov one-sample test was used as a means of determining if it was reasonable to assume that the compression ratio, the recompression ratio, and the coefficient of secondary compression were normally distributed. It was found in each case that normal distributions were accurate at the five percent significance level, and it was concluded that $C_{\varepsilon c}$, $C_{\varepsilon r}$, and C_α could reasonably be considered to be normally distributed. These compressibility variables were treated as random variables in the analyses of settlement described subsequently.

Considerable experience in the San Francisco Bay area has shown that calculated values of settlement are in better agreement with observed rates of settlement when values of C_v are determined using Taylor's square root of time method instead of Casagrande's method. Therefore, Taylor's method was used in the settlement rate analyses described in subsequent sections. However, there was not a significant difference between these two values of C_v.

The variation observed in the measured values of C_v might contribute significantly to differences in the time rate of settlements from place to place. However, after analyzing a number of case histories in which the calculated and measured time rates of settlement were in good agreement, it was decided to treat C_v as a deterministic variable. It can be seen from Fig. 3 that the time rates of settlement in the crusted area are similar even though the average distance between the settlement plates is 500 ft (150 m). In addition, the settlement rates in the non–crusted area (Fig. 4) are nearly identical even though the average distancce between the settlement plates is 680 ft (210 m). Therefore, variations of C_v within each area did not affect the overall time rate of settlement. It is anticipated that C_v ranges over its full range of values within distances of one to six inches (2.5 to 15.2 cm), and thus a deposit many feet thick may be accurately characterized by an average value.

VARIABILITY OF PRECONSOLIDATION PRESSURES

Preconsolidation pressure (p_p) is an extremely important factor in determining the magnitude of consolidation settlement. Since C_{sr} (applicable below the preconsoldation pressure) is only about one–tenth as large as C_{sc} (applicable above the preconsolidation pressure), changes in the value of p_p have the potential to cause as much as a ten–fold increase in the magnitude of the consolidation settlement.

In areas like the crusted area at Bay Farm Island, the thickness of the crust and the magnitude of the preconsoliclation pressures near the ground surface vary considerably from place to place. Measured values of p_p for samples obtained from the crusted area prior to filling are shown in Fig. 6. It can be seen that there is considerable scatter in the near–surface values, and that below about six feet (1.8 m) the values line up along a line parallel to the effective overburden pressure line, with a horizontal offset of 280 psf (13.4 kN/m^2). Based on the results shown in Fig. 6 it was concluded that the large values of p_p near the surface were due to crust formation, and that the offset at greater depth was due to aging effects of the type described by Bjerrum (1973). Values of p_p for samples obtained from the non–crusted area prior to filling plotted along the aging line and showed no evidence of a crust.

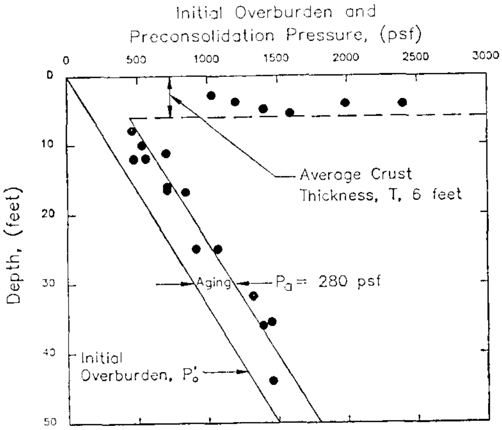


Fig. 6. Initial overburden pressure and preconsolidation pressure in crusted area at Bay Farm Island

The scatter in the near–surface values of p_p are believed to be the most important cause of the variatons in settlement in the crusted area at Bay Farm Island. The basic problem in treating p_p as a random variable is the fact that values of p_p vary significantly with depth, and it is therefore not possible to simply calculate their average and standard devitaion. To develop a statistical measure of how values of p_p vary within the crust zone, a parameter called the "radius of preconsolidation" was devised. Denoted as R, the radius of preconsolidation affords a means of developing a simple statistical measure of the degree of spatial variation in p_p within the crust.

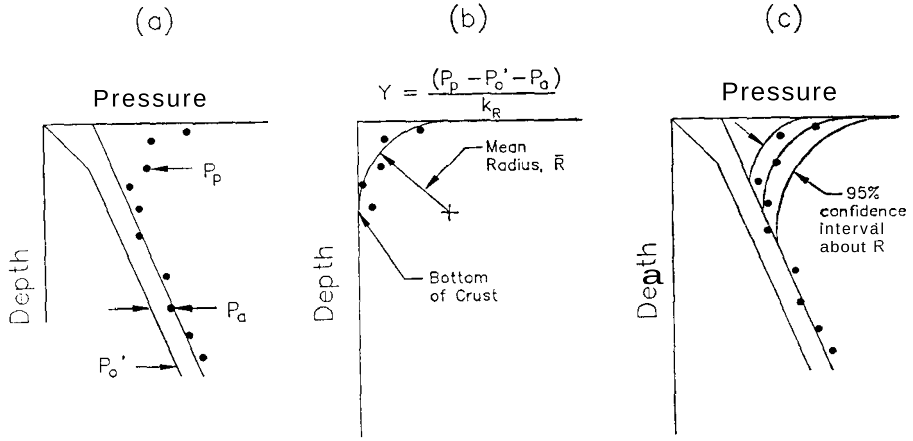


Fig. 7. 95% confidence interval on preconsolidation profile: (a) Typical pressure profile; (b) Mean radius of preconsolidation; (c) 95% confidence interval about mean radius

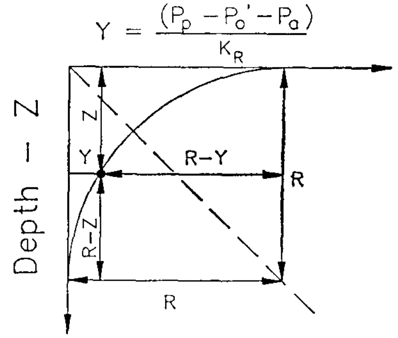
The concept of the radius of preconsolidation is based on the following notion: if the quantity $(p_p - p_o' - p_a)$ = (preconsolidation pressure–effective overburden pressure–aging pressure) is plotted versus depth, the variation with depth can be described by a circle. This is illustrated in Fig. 7 (b), where the values of $(p_p - p_o' - p_a)$ are divided by a factor termed k_R and plotted versus depth.

Once the value of k_R has been determined for a particular site (the procedure is explained below) values of R can be calculated using each value of p_p and the corresponding depth, Z. These values of R are calculated by repeated trials using this implicit expression:

$$R = \frac{(p_p - p_o' - p_a)}{k_R} + \sqrt{2RZ - Z^2} \qquad (1)$$

Eq. (1) is derived from the equation of a circle as shown in Fig. 8. The engineering units of k_R are Force/Length^3. Therefore, dividing $(p_p - p_o' - p_a)$ by k_R results in a dimensionally consistent relationship between Y and depth. Each value of R represents the radius of a circle that would pass through the point $[(p_p - p_o' - p_a), Z]$ in Fig. 7(b).

Being a single quantity, the average value of R and its standard deviation can be determined, and these values can be used conveniently to estimate practical upper and lower limits of preconsolidation pressure



Using the equation of a circle,

$$(R-Z)^2 + (R-Y)^2 = R^2$$

expanding the squares, and solving the quadratic equation,

$$Y = \frac{2R - \sqrt{4R^2 - 4R^2 + 8RZ - 4Z^2}}{2}$$

which reduces to

$$Y = R - \sqrt{2RZ - Z^2}$$

then substituting for Y,

$$Y = \frac{(P_p - P_o' - P_a)}{K_R} = R - \sqrt{2RZ - Z^2}$$

or

$$R = \frac{(P_p - P_o' - P_a)}{K_R} + \sqrt{2RZ - Z^2} \quad \ldots (1)$$

Fig. 8. Derivation of radius of preconsolidation

profiles within the crust zone. Assuming that the values of R are normally distributed, 95% of the values should fall within plus or minus 1.96 standard deviations from the average. Thus if two preconsolidation pressure profiles are considered, one corresponding to R minus 1.96 standard deviations, and one corresponding to R plus 1.96 standard deviations, about 95% of all measured values of p_p should fall within these bounds. This is illustrated in Fig. 7(c).

The value of parameter k_R for a given site is calculated by rearranging equation (1) into the following form:

$$k_R = \frac{(p_p - p_o' - p_a)_{\text{ave}}}{(T - \sqrt{2TZ_{\text{ave}} - (Z_{\text{ave}})^2})} \qquad (2)$$

where T is the crust thickness, and the subscript "ave" indicates average values of the particular property. The crust thickness is determined by visual inspection of the preconsolidation profile.

An example of this procedure for Bay Farm Island is shown in Table 2. In the crusted area, six samples had values of p_p that significantly exceeded the aging pressure profile, and these were used in calculating the statistics shown in Table 2. First, the value of k_R was calculated using equation (2). Then, using this value of k_R, values of R were calculated for each sample individually.

Table 2. Determination of radius of preconsolidation for crusted area at Bay Farm Island

Depth, Z (ft)	p_o' (psf)	p_p (psf)	$(p_p - p_o' - p_a)$ (psf)	R (ft)
2.8	84	1030	670	4.0
3.8	115	1200	800	5.3
4.2	127	2400	2000	6.7
5.5	166	1600	1140	7.2
4.0	127	2000	1593	6.0
5.0	61	1400	1059	6.9
Averages = 4.2 *feet*			1210 psf	6.0 *feet*

$$k_R = \frac{(p_p - p_o' - p_a)_{\text{ave}}}{(T - \sqrt{2TZ_{\text{ave}} - Z_{\text{ave}}^2})} = \frac{1210\ \text{psf}}{(6' - \sqrt{2(6')4.2' - (4.2')^2})}$$

$$= 4378_{\text{pcf}}$$

$\bar{R} = 6.0$ *feet*

$S(\bar{R}) = 1.2$ *feet*

$R_{95\%} = \bar{R} \pm 1.96(S(\bar{R})) = 6.0' \pm 1.96(1.2') = 8.4$ and 3.6 feet

As shown in Table 2, the average value of R, i.e., $\bar{R}$, is 6.0 ft. (1.8 m), and the standard deviation is 1.2 ft. (0.4 m). Thus the 95% confidence limit values of R are 6.0−2.4=3.6 ft. (1.1 m) and 6.0+2.4=8.4 ft. (2.6 m). These values, together with $k_R = 4378$ lb per cu. ft. (687.8 kN/m^3) and Eq. (1), were used to estimate practical upper and lower limit preconsolidation pressue profiles. Then 95% confidence limits for settlement were estimated using these preconsolidation pressure profiles. Allowance for the effects of variation in compressibility were also made, as explained below. The use of six data points to compute the mean and standard deviation of R is not theoretically desirable. However, the use of the radius of preconsolidation at least provides a systematic approach to quantifying the variability of p_p using the limited data that is usually available.

ESTIMATING 95% CONFIDENCE LIMITS FOR SETTLEMENT

The concept discussed previously can be used to estimate 95% confidence limits for settlement. For practical purposes these provide upper and lower bounds of estimated settlement. The procedures for estimating these bounds are explained in the following paragraphs.

Effects of Preconsolidation Pressure.

As discussed previously, preconsolidation pressure profiles can be described in terms of R. Upper and lower bound estimates of consolidation settlement are made using conventional procedures, one (ΔH_{cU}) using the preconsolidation pressure profile corresponding to $[\bar{R} - 1.96 * S(\bar{R})]$, and the other ($\Delta H_{cL}$) using the preconsolidation pressure profile corresponding to $[\bar{R} + 1.96 * S(\bar{R})]$. For this purpose the clay is divided into a number of sublayers. The values of consolidation settlement, ΔH_c are calculated using the conventional equation:

$$\Delta H_c = \sum_{i=1}^{N} \left\{ H_i \left[\bar{C}_{\varepsilon r} \log \frac{p_{pi}}{p_o'{}_i} + \bar{C}_{\varepsilon c} \log \frac{p_f'{}_i}{p_{pi}} \right] \right\} \qquad (3)$$

where N =number of sublayers,
i =sublayer number,
H_i =sublayer thickness,
p_o' =effective overburden pressure,
p_f' =effective final pressure,
$\bar{C}_{\varepsilon r}$=average value of $C_{\varepsilon r}$, and
$\bar{C}_{\varepsilon c}$=average value of $C_{\varepsilon c}$.

To calculate the lower bound of consolidation settlement, ΔH_{cL}, values of p_p corresponding to $[\bar{R}+1.96*S(\bar{R})]$ are used. To calculate the upper bound, ΔH_{cU}, values of p_p corresponding to $[R-1.96*S(\bar{R})]$ are used. These calculations are performed using the average values of $C_{\varepsilon r}$, and $C_{\varepsilon c}$, and p_o', and p_f' at the middle of each sublayer.

Effects of Scatter in *Compressibility.*

Variations in the average values of $C_{\varepsilon r}$ and $C_{\varepsilon c}$ result in variations in the magnitude of consolidation settlement. The variance in settlement arising from variations in the values of the random variables $C_{\varepsilon r}$ and $C_{\varepsilon c}$ can be derived as explained by Benjamin and Cornell (1970) or by Ang and Tang (1975). The expression for the variance can be shown to be:

$$\left\{\begin{aligned} S(\mathrm{AH,})^2 = \sum_{i=1}^{N} \Big\{ & (H_i)^2 \log^2\left(\frac{p_{pi}}{p_o'{}_i}\right) * S(C_{\varepsilon r})^2 \\ & + (H_i)^2 \log^2\left(\frac{p_f'{}_i}{p_{pi}}\right) * S(C_{\varepsilon c})^2 \\ & + 2(H_i)^2 \log\left(\frac{p_{pi}}{p_o'{}_i}\right) \log\left(\frac{p_f'{}_i}{p_{pi}}\right) * C(C_{\varepsilon c}, C_{\varepsilon r}) \Big\} \end{aligned}\right. \tag{4}$$

where $S(C_{\varepsilon r})^2$ =variance of $C_{\varepsilon r}$,
$S(C_{\varepsilon c})^2$ =variance of $C_{\varepsilon c}$, and
$C(C_{\varepsilon r}, C_{\varepsilon c})$ =covariance of $C_{\varepsilon c}$ and $C_{\varepsilon r}$.

The variances of $C_{\varepsilon r}$ and $C_{\varepsilon c}$ are calculated using these expressions:

$$S(C_{\varepsilon r})^2 = \frac{1}{(n-1)} \sum_{i=1}^{n} (C_{\varepsilon ri} - \bar{C}_{\varepsilon r})^2, \text{ and} \tag{5}$$

$$S(C_{\varepsilon c})^2 = \frac{1}{(n-1)} \sum_{i=1}^{n} (C_{\varepsilon ci} - \bar{C}_{\varepsilon c})^2, \tag{6}$$

where n =number of measurements of $C_{\varepsilon r}$ and $C_{\varepsilon c}$,
$C_{\varepsilon ri}$=value of $C_{\varepsilon r}$ for test i, and
$C_{\varepsilon ci}$=value of $C_{\varepsilon r}$ for test i.

The covariance of $C_{\varepsilon r}$ and $C_{\varepsilon c}$ is calculated using the expression:

$$C(C_{\varepsilon r}, C_{\varepsilon c}) = \frac{1}{n} \sum_{i=1}^{n} (C_{\varepsilon ri} - \bar{C}_{\varepsilon r}) * (C_{\varepsilon ci} - \bar{C}_{\varepsilon c}). \tag{7}$$

With 95% confidence, the magnitude of the variation in settlement caused by variations in the average values of compressibility can be expressed as:

$$\Delta_{95} = 1.96 * \sqrt{S(\Delta H_c)^2} \tag{8}$$

where $S(\Delta H_c)^2$=variance of consolidation settlement from Eq. (4).

Effects of Scatter *in Secondary Compression*

Secondary compression settlements can be estimated using the following equation:

$$\Delta H_{sc} = H\bar{C}_{\alpha} \log\left(\frac{t}{t_{is}}\right) \tag{9}$$

where ΔH_{sc}=settlement due to secondary compression,
H =layer thickness,
C_{α} =average value of coefficient of secondary compression,
t =time (must be greater than t_{is}), and
t_{is} =time of initiation of secondary compression.

There are considerable difference of opinion concerning the time at which secondary compression begins. For simplicity, it was assumed that secondary compression starts at a degree of consolidation of 90%. Therefore, secondary compression is added to any clay layer with a degree of consolidation greater than or equal to 90%. Javete and Duncan (1983) showed that t_{is} logically varies with layer thickness. On this basis they found the following values of t_{is} for San Francisco Bay Mud:

Layer Thickness (ft)	Value of t_{is} (years)
10	1.1
20	4.1
30	8.7
40	14.9
50	22.6

These values were used in calculating the Bay Mud settlements discussed.

Practical upper and lower limits of secondary compression settlement can be calculated by considering possible variations in the average value of C_α. Two calculations can be done using values of C_α that are $(1.96*S(\bar{C}_\alpha))$ above and below $\bar{C}_\alpha$, where $\bar{C}_\alpha$ is the average value of C_α, $S(C_\alpha)$ is the standard deviation of C_α, and $S(\bar{C}_\alpha)$ is the standard deviation of $\bar{C}_\alpha$, given by the following expression:

$$S(\bar{C}_\alpha)=S(C_\alpha)/\sqrt{n} \qquad (10)$$

Range of Settlement at any *Time*.

The 95% confidence limits for settlement can be calculated using the average values and deviations from the average discussed previously. The lower bound (with 95% confidence) can be estimated using the expression :

$$\varDelta H_{tL}=[\varDelta H_{cL}-\varDelta_{95}]*U+\varDelta H_{scL} \qquad (11)$$

where $\varDelta H_{tL}$ =lower bound settlement at time t,
$\varDelta H_{cL}$ =value of $\varDelta H_c$ calculated using upper bound values of p_p,
$\varDelta_{95}$ =95% confidence limit on variation of settlement,
U =degree of consolidation at time t, and
$\varDelta H_{scL}$=lower bound of secondary compression settlement at time t.

The value of $\varDelta H_{cL}$ in Eq, (11) is calculated using the upper bound values of preconsolidation pressure, $[\bar{R}+1.96*S(R)]$, and the value of $\varDelta H_{scL}$ is calculated using the lower bound value, $[\bar{C}_\alpha-1.96*S(\bar{C}_\alpha)]$, of C_α.

The upper bound settlement (also with 95% confidence) can be estimated using the expression :

$$\varDelta H_{tU}=[H_{cU}+\varDelta_{95}]*U+\varDelta H_{scU} \qquad (12)$$

where $\varDelta H_{tU}$ =upper bound settlement at time t,
$\varDelta H_{cU}$ =value of $\varDelta H_{cU}$ calculated using lower bound values of p_p, and
$\varDelta H_{scU}$=upper bound of secondary compression settlement at time t.

The value of $\varDelta H_{cU}$ in Eq. (12) is calculated using the lower bound values of preconsolidation pressure, $[\bar{R}-1.96*S(\bar{R})]$, and the value of $\varDelta H_{scU}$ is calculated using the upper bound value, $[\bar{C}_\alpha+1.96*S(C_\alpha)]$, of C_α.

The values of $\varDelta H_{tU}$ and H_{tL} provide upper and lower bound estimates of settlement within which about 95% of the observed settlements should fall. The method has been applied to five fill sites in the San Francisco Bay Area (Bay Farm Island, Redwood Shores, the TWA site at San Francisco Airport, and the Folgers Coffee building site), and two sites near Boston (the MIT test embankment and the Northeast Test Embankment). In all of these cases the method was found to provide realistic upper and lower bounds for settlement, neither much narrower nor much wider than the range of observed settlements. Two of the cases studied (Bay Farm Island and the Northeast Test Embankment) are described in subsequent sections of this paper.

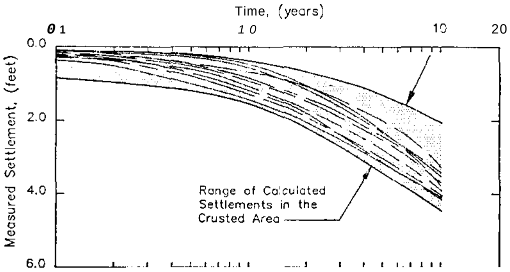


Fig. 9. Observed and calculated range of settlements for crusted area at Bay Farm Island

The method of estimating a range of settlements on clay described above has been found to provide a more complete and more realistic analysis of settlements on clays. In addition, the method requires no more testing than is done for conventional settlement analyses and only slightly more computation.

APPLICATION TO BAY FARM ISLAND

The settlement Bay Farm Island provided the impetus for this study, and this site was the first to which the methods were applied. The properties summarized in Table 1 and the preconsolidation pressures shown in Fig. 6 were used to estimate 95% confidence limits for the settlements in the crusted area. It can be seen from Fig. 9 that the calculated range of settlements encloses approximately 95% of the measurements, and that the calculations provide a good approximation of the breadth in scatter actually seen in the field.

APPLICATION TO THE MIT AND NORTHEAST TEST EMBANKMENTS

Lambe (1973) described the MIT and the Northeast Test Embankments in his Rankine Lecture. The embankments are located on and near a section of Interstate Highway 1–95 that passes through a tidal marsh north of Boston, Massachusetts.

At both sites the Boston Blue Clay is overlain by approximately 4 ft (1.2 m) of sand and 7 ft (2.1 m) of peat. The ground surface, i. e., the top of the peat, is located at elevation +5 ft (1.5 m). At the Northeast Test Embankment the Boston Blue Clay extends to an elevation of approximately —100 ft. (—30.5 m), whereas at the MIT Test Embankment the Clay extends to an elevation of —150 ft. (—45.8 m). Beneath the Boston Blue Clay is a layer of glacial till, and beneath that, shale. Laboratory tests showed that the Boston Blue Clay at both sites has a very thick and variable desiccated crust. The desiccated crust extends from an elevation of —6 ft (1.8 m) to elevation —70 ft (21.4 m). The overconsolidation ratio decreases from approximately 9.0 at the top of the crust to 1.0 at an elevation of —70 ft (21.4 m).

A very puzzling aspect of the performance of the Northeast Embankment is the fact that three settlement plates, about 100 ft. (30.5 m) apart on the crest of the embankment, settled by amounts that differed very significantly. During a period of 15 years after construction of the embankment, Plate No. 1 settled about 3.0 ft. (0.9 m), Plate 2 settled about 4.2 ft. (1.3 m), and Plate No. 3 settled about 5.6 ft. (1.7 m). In his Rankine Lecture, Lambe stated:

> "... considerable time and effort were expended in an unsuccessful attempt to explain the different settlement behaviour at the three plates."

He concluded that the most probable reason for the large differences in settlement at the three plates was differences in the compressibility of the foudation soils, but noted that this could not be proved. The authors' have found that the probability concepts and method of settlement analysis described in this paper provide **a** very useful approach to understanding these seemingly anomalous settlement differences that are so difficult to explain from a deterministic point of view.

The methods described previously were applied to the settlement of the Northeast Test Embankment, with the results shown in Fig. 10. The immediate settlement was estimated using the undrained shear strength values presented by Lambe (1973) and the procedure proposed Skempton and Bjerrum (1957). The applied stress was calculated using

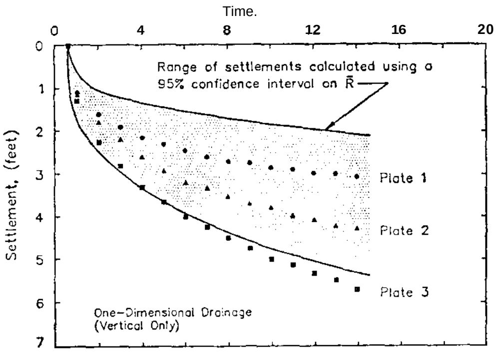


Fig. 10. Measured and calculated settlements at NE test embankment-one dimensional drainage

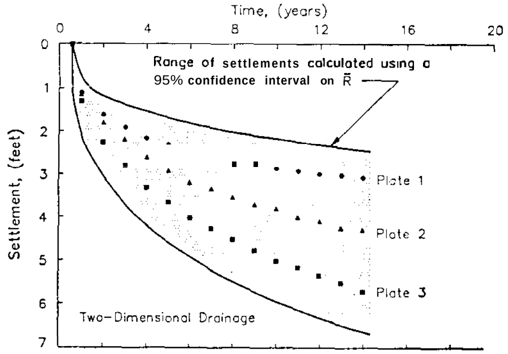


Fig. 11. Measured and calculated settlements at NE test embankment-two Dimensional drainage

Westergaard's stress distribution theory and a detailed record of the fill placement. It can be seen that the range of calculated settlements is about the same as the range of measured settlements. However, the rate of calculated settlements is slightly slower than the rate at which the settlements actually occurred.

Since the measured and calculated time curves indicate a similar value of C_v and the width of the embankment (100 ft., 30.5 m) was not very great compared to the depth of the clay on which it rested, it was considered that the field rate of settlement might be faster as a result of lateral drainage. Accordingly, a second analysis was performed using values of degree of consolidation determined using the adjustment factors for two-dimensional drainage suggested by Lambe et al. (1972). The results of this second analysis are shown in Fig. 11. It may be seen that the measured settlements all fall within the calculated range. On this basis, it appears likely that the very large variations in measured settlements at the Northeast Test Embankment were due to spatial variations in preconsolidation pressure in the desiccated crust. A similar analysis was performed for the settlements at the MIT Test Embankment, with essentially similar results.

SUMMARY AND CONCLUSIONS

The method described in this paper can be used to estimate practical upper and lower bounds for settlements on clay deposits which have a preconsolidated crust. It uses conventional settlement analysis procedures and simple probability concepts to estimate 95% confidence limits for settlements. Variations in compressibility are accounted for by the standard deviations of $C_{\epsilon r}$ and $C_{\epsilon c}$, and variations in preconsolidation pressure are accounted for by the standard deviation of the "radius of preconsolidation," a parameter developed to make it possible to statistically characterize variations in preconsolidation pressure in the crust zone. Analysis of the settlement at five sites on San Francisco Bay Mud and two sites on Boston Blue Clay has shown that where desiccated crust has developed on the clay, variations in preconsolidation pressure are the most important cause of differential settlements.

Comparison of calculated ranges of settlement with measured ranges of settlement at seven different sites has shown that the method is practical and effective. It requires no more testing than is done for conventional analysis of settlements on clay. Unlike conventional settlement analyses, however, it provides a means for estimating the likelihood that the settlement will vary from the mean by a given amount due to random variations in soil properties.

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NOTATION

The following symbols are used in the paper:

C_α =coefficient of secondary compression

$\overline{C_\alpha}$ =average value of C_α

$C_{\varepsilon c}$ =compression ratio

$\overline{C_{\varepsilon c}}$ =average value of $C_{\varepsilon c}$

$C_{\varepsilon r}$ =recompression ratio

$\overline{C_{\varepsilon r}}$ =average value of $C_{\varepsilon r}$

C_v =coefficient of consolidation

$C(C_{\varepsilon c},C_{\varepsilon r})$ =covariance of $C_{\varepsilon r}$ and $C_{\varepsilon c}$

H =sublayer thickness

k_R =radius of preconsolidation scaling factor

p_a =soil aging pressure

p_f' =final effective applied pressure

p_0' =initial effective overburden pressure

p_p =effective preconsolidation pressure

R =radius of preconsolidation

$\bar{R}$ =average value of radius of preconsolidation

$S(C_\alpha)$ =standard deviation of C_α

$S(C_{\varepsilon c})^2$ =variance in $C_{\varepsilon c}$

$S(C_{\varepsilon r})^2$ =variance in $C_{\varepsilon r}$

$S(\Delta H_c)^2$ =variance in consolidation settlement

T =observed crust thicknes

t =time

t_{is} =secondary compression initiation time

U =degree of consolidation

Z =sample depth

Z_{ave} =average sample depth

ΔH_c =consolidation settlement

ΔH_{cL} =lower bound consolidation settlement

ΔH_{cU} =upper bound consolidation settlement

ΔH_{sc} =secondary compression settlement

ΔH_{scL} =lower bound secondary compression settlement

ΔH_{scU} =upper bound secondary compression settlement

ΔH_{tL} =lower bound settlement at time t

ΔH_{tU} =upper bound settlement at time t

Δ_{95} =95% variation in consolidation settlement