SOILS AND FOUNDATIONS Vol. 39, No. 5, 000–000, Oct. 1999
Japanese Geotechnical Society

RELIABILITY IN BACK ANALYSIS OF SLOPE FAILURES

WILSON H. TANG[i], TIMOTHY D. STARK[ii] and MAURICIO ANGULO[iii]

ABSTRACT

The advantages and limitations of using a back analysis of slope failures to evaluate soil shear strength are discussed. A methodology is presented herein that allows the implied level of reliability associated with soil shear strength parameters back calculated from slope failures to be estimated. A reliability approach is also used to estimate the probability of failure for a given limit equilibrium slope stability method, design factor of safety, and combination of back calculated Mohr-Coulomb shear strength parameters, c' and ϕ'. The methodology is illustrated using 39 landslides in the Orinda Formation in the San Francisco Bay area. The impact of additional case histories in the same geologic setting, i.e., a larger data set, on the required design factor of safety for a given probability of failure is also investigated.

Key words: landslide, shear strength, slope stability, statistical analysis (**IGC**: D6/E6/F6/H9)

INTRODUCTION

The analysis of slope stability involves many areas of uncertainty, including computational accuracy, soil unit weight, slope geometry, pore-water pressures, and soil shear strength. (Soil shear strength is usually represented by the effective stress Mohr-Coulomb shear strength parameters, cohesion, c', and angle of internal friction, ϕ'.) Extensive research has been conducted to evaluate the computational accuracy of two-dimensional limit equilibrium slope stability methods, e.g., Wright et al. (1973), Fredlund and Krahn (1977), Duncan and Wright (1980), Leshchinsky (1990), and Duncan (1992). This research has shown stability methods that satisfy all conditions of equilibrium (horizontal and vertical force equilibrium and moment equilibrium) result in a factor of safety with an accuracy of ± 5 percent (Duncan, 1992). As a result, stability methods that satisfy all conditions of equilibrium, e.g., Janbu (1968), Morgenstern and Price (1965), and Spencer (1967), can be considered to yield an accurate estimate of the factor of safety.

Because the uncertainty in computational accuracy is small for methods that satisfy all conditions of equilibrium, it is necessary to consider possible uncertainties in the other previously mentioned parameters. Soil unit weight can be readily measured in the laboratory, slope geometry can be ascertained via elevation surveys and subsurface techniques, and pore-water pressures can be estimated from boreholes and/or piezometer installations. However, a large source of uncertainty can be introduced during the selection of the soil shear strength parameters. Soil shear strength parameters for slope design are usually estimated from laboratory testing. However, laboratory measured shear strengths involve uncertainties because of the need to obtain a representative sample of the materials involved in the potential failure surface and to simulate the field conditions existing in the slope. The field conditions that must be reproduced in the laboratory include effective normal stress acting on the failure surface, pre-existing, if any, shear deformation prior to failure, drainage during shear, mode of shear, shear displacement rate, and formation of a shear surface. Therefore, estimating the soil shear strength can lead to significant uncertainty in slope stability computations.

Determining soil shear strengths by back analysis avoids many of the problems associated with laboratory testing, and is widely used especially in connection with landslide repairs. Back analysis is an effective method for incorporating important factors that may not be well represented in laboratory samples, such as the structural fabric of the soil, nonhomogeneity, influence of fissures on soil shear strength, and the effects of pre-existing shear planes within the soil mass. A back analysis assumes the original slope geometry and a factor of safety equal to unity to estimate the soil shear strength that was mobilized for the failure to have occurred consistent with the two-dimensional limit equilibrium model, e.g., Spencer's (1967) method, adopted for the analysis. This back calculation yields the soil shear strength that was

[i] Professor and Head, Department of Civil and Structural Engineering Hong Kong University of Science and Technology, Hong Kong.
[ii] Associate Professor, Department of Civil Engineering 2217 Newmark Civil Engineering Lab (MC-250) University of Illinois 205 N. Mathews Ave. Urbana, IL 61801, USA.
[iii] Geotechnical Engineer, Harza Engineering Company, Inc. 233 S. Wacker Dr. Chicago, IL 60606, USA.
Manuscript was received for review on July 13, 1998.
Written discussions on this paper should be submitted before May 1, 2000 to the Japanese Geotechnical Society, Sugayama Bldg. 4F, Kanda Awaji-cho 2-23, Chiyoda-ku, Tokyo 101-0063, Japan. Upon request the closing date may be extended one month.

mobilized along the entire length of the failure surface in the suspect material at the time of failure.

In most cases, the information regarding the conditions under which a landslide occurred is limited. This lack of complete information reduces the reliability of the back calculated shear strengths. However, useful values of c' and ϕ' can be obtained if the information concerning the failure conditions is not extremely deficient and/or reasonable assumptions can be made that are supported by local experience and good judgment. These back-calculated values can be used to analyze the stability of other slopes in the same geologic formation. When many slides in the same formation are back analyzed, a measure of the reliability of the back calculated shear strength parameters can be developed. Duncan and Stark (1992) used back analysis of 39 landslides in the Orinda Formation in Contra Costa County, California, to illustrate the method of calculating the reliability of shear strength parameters that were back calculated from multiple slides in the same geologic formation.

ORINDA FORMATION

The Pliocene Orinda Formation includes conglomerate, sandstone, siltstone, and claystones and is highly prone to landslides. Radbruch and Weiler (1963) studied 195 landslides that occurred in a two-year period in the Orinda Formation. These landslides occurred in a 22 square kilometer area just east of the Berkeley hills in the San Francisco Bay area. The study area consists of northwest-southeast trending parallel folds that are offset by several faults. The relief in the area is as large as 185 m and the soil cover is sparse.

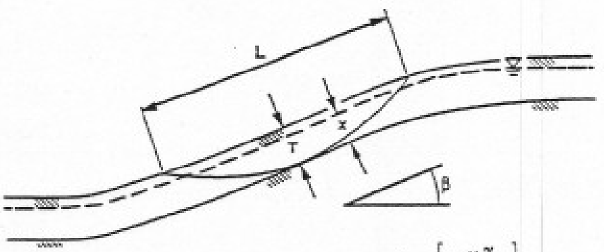
In most locations, the weathered rocks of the Orinda Formation are exposed on the hillsides. Weathering extends to a depth of one to seven meters. The thickest and thinnest zones of weathered material usually occur at the bottom and top, respectively, of the slope. The weathered materials of the Orinda Formation have low shear strength. Although they are rocks from a geologic standpoint, they behave as a soil or weathered rock. The weathered materials are primarily cohesive and classify as clays of low to medium plasticity (CL) according to the Unified Soil Classification System. Typical values of the Atterberg limits for the weathered materials are a liquid limit of 30 to 50 and a plastic limit of 15 to 25. The low shear strength of the weathered materials and the build-up of pore-water pressure at the interface between the weathered and unweathered materials are the causes of most of the landslides. The landslides occur on natural slopes inclined at 20 degrees or more, and rarely on slopes flatter than 20 degrees. The build-up of pore-water pressure at the weathered/unweathered bedrock interface usually occurs due to surface infiltration and thus most of the failures occur during the rainy/wet months of the year. The precipitation infiltrates the weathered materials and percolates down to the lower permeability unweathered material. This results in pore-water pressures building up at the top of the unweathered material and causing a reduction in the effective stress and factor of safety at the interface. These pore-water pressures cannot drain rapidly because the weathered materials consist of clayey/cohesive soil. This build-up in pore-water pressure usually results in the failure surface extending to the interface between the weathered and unweathered materials. This will facilitate the determination of the failure surface geometry and soil shear strength in the back analysis.

Back Analysis Procedure

Of the 195 landslides that were recorded by Radbruch and Weiler (1963), a total of 39 were used by Duncan and Stark (1992) for back analysis. These include 24 slides in natural slopes, 10 slides in cut slopes, and 5 slides in slopes that were constructed by placing fill derived from the Orinda Formation on the weathered materials. A slope failure provides only one piece of information, i.e. the factor of safety is equal to unity. Therefore, the soil shear resistance can be varied to achieve a factor of safety equal to unity. The shear resistance is usually represented by a combination of c' and ϕ'. Determining the appropriate combination of c' and ϕ' is complicated because the magnitudes of c' and ϕ' control the location of the failure surface. For example, in homogeneous slopes with soils exhibiting a ϕ' greater than zero, the critical failure surface usually passes through the toe of the slope (Duncan, 1996). However, increasing the value of c' with a constant value of ϕ' will cause the critical failure surface to extend deeper into the deposit even though it still exits through the toe. Therefore, the location of the failure is controlled by the combination of c' and ϕ'.

If the position of the failure surface is controlled by the location of strong or weak layers within the slope, the shear resistance can be calculated because the location of the failure surface is known and not a function of the combination of c' and ϕ'. If the required shear resistance and the corresponding effective stress are known, a combination of c' and ϕ' can be selected to represent the required shear resistance on the failure surface. As discussed previously, the failure surface in the Orinda Formation is usually located at the boundary between the weathered and unweathered bedrock, which is typical of colluvial slopes. Since the value of ϕ' can be readily obtained (Table 1), it is preferable to assume the value of ϕ' and back calculate the value of c' that is required to represent the shear resistance for a factor of safety of unity and the observed failure surface. Duncan and Stark (1992) calculated the value of c' for a factor of safety of unity for each of the Orinda Formation slopes using assumed values of ϕ' ranging from 20 degrees to 40 degrees.

Duncan and Stark (1992) estimated the range of ϕ' using an empirical correlation (Table 1) derived from Ladd et al. (1977) and Mitchell (1993) and the value of plasticity index. This resulted in 24 values of c' being back calculated for each assumed value of ϕ' for the natural slopes.



$$A = \left[1 - \frac{X}{T}\frac{\gamma_w}{\gamma_m}\right]$$

$$F = C_1 A \frac{\tan \phi'}{\tan \beta} + C_2 B \frac{c'}{\gamma_m H} \qquad B = \frac{1}{\cos^2\beta \tan \beta}$$

$$C_1 = \left[1 + 36 \sin \beta \left(\frac{T}{L}\right)^2\right]$$

$$c' = \frac{\gamma_m H}{C_2 B}\left[F - C_1 A \frac{\tan \phi'}{\tan \beta}\right] \qquad C_2 = \left[1.5 + 10\left(\frac{T}{L}\right)^2\right]$$

$$H = \frac{T}{\cos \beta}$$

F = factor of safety
ϕ' = effective stress angle of internal friction
c' = effective stress cohesion intercept
γ_m = moist (total) unit weight of soil
γ_w = unit weight of water
T = thickness of layer containing slide (measured normal to ground surface)
X = distance from phreatic surface to base of layer (normal to ground surface)
H = vertical distance from top of layer to base
β = slope angle

Fig. 1. Analysis of rotational slides on long slopes (from Duncan and Stark, 1992)

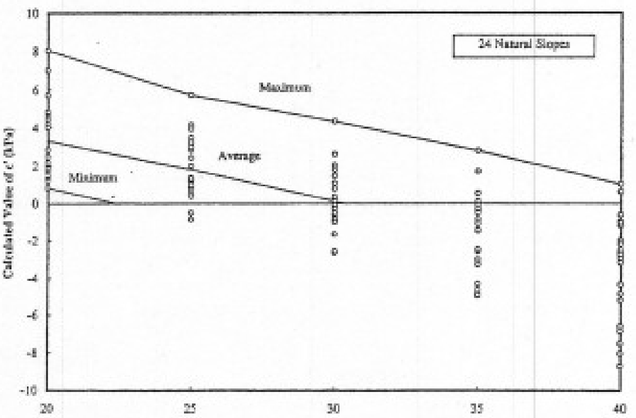


Fig. 2. Variation of calculated value of c' with assumed value of ϕ' for failures on 24 natural slopes in the Orinda Formation (after Stark and Duncan, 1992)

The procedure for the back analysis, described in Duncan and Stark (1992), uses a circular failure surface and the equations shown in Fig. 1 to calculate c' for a given ϕ' and a factor of safety equal to unity. The equations shown in Fig. 1 were developed by analyzing a number of rotational slides on long slopes using Bishop's Modified Method (Bishop, 1955) and relating the results to the expressions for the factor of safety from an infinite slope analysis (Duncan and Stark, 1992). A circular failure surface was used because it is considered to be more realistic than the infinite slope model, which neglects end effects at the head and the toe of the slide, and because it represents field observations. Consideration of the end effects results in values of c' that are somewhat smaller, for the same value of ϕ', than would have been calculated if the infinite slope mechanism had been used in the back analysis. This rotational model was adopted for all of the natural, cut, and fill slope cases that were analyzed during this study. It should also be noted that the circular failure surface extends down to the weathered/unweathered interface and has a long radius to represent a long shallow failure surface.

Table 1. Typical values of ϕ' for the fully softened and residual strength conditions (after Ladd et al., 1977 and Mitchell, 1993)

Plasticity Index	Value of ϕ' (degrees)	
	Fully Softened	Residual
0–10	30–40	18–30
10–20	25–35	12–25
20–40	20–30	10–20
40–80	15–25	7–15

Duncan and Stark (1992) showed the variation of back-calculated values of c' with the assumed values of ϕ' for the 24 landslides in natural slopes at a factor of safety of unity (Fig. 2). It can be seen that 24 values of c' are greater than zero for a ϕ' of 20 degrees. However, as ϕ' increases the required value of c' must decrease to maintain a factor of safety of unity. This results in some negative values of c' for values of ϕ' greater than 25 degrees. Since a negative value of c' has no physical meaning, the trend lines in Fig. 2 terminate at a value of c' equal to zero. However, the data points for the 24 slopes are shown for each value of ϕ' for completeness purposes even though some of the values of c' are negative for ϕ' greater than 25 degrees.

Duncan and Stark (1992) recommend for design using the average value of c' with the corresponding value of ϕ' and to ensure that the calculated factor of safety is larger than the maximum factor of safety shown in Fig. 3 for that combination of c' and ϕ'. Thus, Duncan and Stark (1992) provide guidance on the best combination of back-calculated c' and ϕ' and the design factor of safety that should be used with that combination. For example, for $\phi'=25$ degrees and $c'=1.9$ kPa (Fig. 2), the design factor of safety should be greater than 1.4 (Fig. 3) for natural slopes in the Orinda Formation. However, the reliability or probability of failure for the design factor of safety and the corresponding combination of c' and ϕ' is not known. It is anticipated that the fewer the number of slope failure considered, e.g. 5 in fill slopes, the less reliable the design.

The objective of this paper is to present a methodology that estimates the implied level of reliability associated with soil shear strength parameters back calculated from slope failures using the analysis shown in Fig. 1 and the corresponding design factor of safety in Fig. 3. The

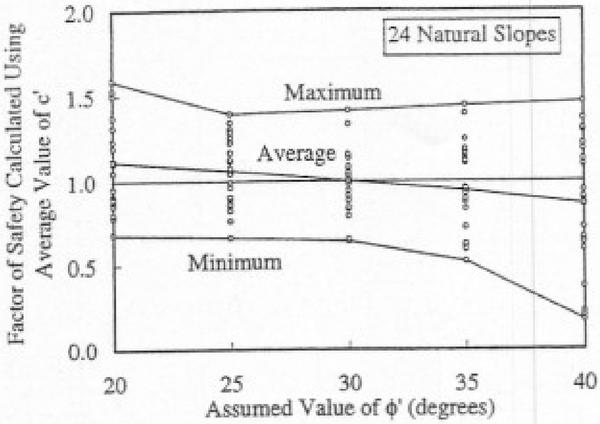


Fig. 3. Variation of factor of safety calculated using the average back calculated value of c' with the corresponding assumed value of ϕ' (from Duncan and Stark, 1992)

methodology presented in this paper is used to estimate the corresponding probability of failure based directly on observed slope performance. Hence, the uncertainty resulting from the discrepancy between field and laboratory performance and the uncertainty associated with the simplified two-dimensional analysis model in Fig. 1 can be avoided.

RELIABILITY CONSIDERATIONS

The maximum values of factor of safety in Fig. 3, and thus the subsequent design factors of safety, are dependent on the specific set of landslide data. For example, Duncan and Stark (1992) used three different types of slopes, namely natural, cut, and fill slopes. The number of landslide case histories is less for the fill (5) and cut (10) slopes relative to that for the natural (24) slopes. As a result, the reliability level associated with the proposed design factor of safety differs among these three slope types. Clearly, another set of landslide data from the region (even for the same number of landslide cases) may reveal a lower or higher value for the maximum factor of safety for a given combination of c' and ϕ'. In fact, if twice the number of landslide cases were available, the maximum factor of safety for this larger set of landslide data could be larger than those shown in Fig. 3. Therefore, the method proposed by Duncan and Stark (1992) of using the observed maximum factor of safety for design cannot guarantee complete safety of future slopes.

Therefore, the main objective of this research was to develop a method for estimating the reliability associated with the proposed design factor of safety obtained from Fig. 3. The proposed method can also be used to estimate how much higher the design factor of safety should be to achieve the desired reliability level, if the reliability of the proposed design factor of safety is too low. Finally, the paper discusses how the design factor of safety can be adjusted to account for the number of case histories that are available in a particular geologic setting.

Reliability Analysis Procedure

Design values of c' were estimated for an assumed value of ϕ' for slopes in the Orinda Formation. For example, the analyzed values of c' are 3.3, 1.9, and 0.1 kPa for design values of ϕ' equal to 20, 25 and 30 degrees, respectively, for natural slopes in the Orinda Formation (Fig. 2). Before proceeding any further, it is important to distinguish design from analysis techniques. In design, some factors that contribute negligibly to slope stability or factor of safety are ignored for the sake of simplicity. However, accounting for all of the factors is important when considering analytical procedures such as reliability. The objective of the proposed reliability method is to account for the majority of the factors influencing slope stability to estimate an accurate valuê of design factor of safety for each slope. This includes accounting for relatively small values of c' such as 3.3, 1.9, and 0.1 kPa as discussed previously. Accounting for small values of cohesion is important because the value of c' directly influences the calculated factor of safety. For example, in Fig. 1 the value of c' is added directly to the factor of safety whereas the tangent of the friction angle is added directly to the factor of safety. The value of cohesion can have an even larger influence on the factor of safety than shown in Fig. 1 when other equilibrium stability methods, e.g., Spencer (1967) and Bishop (1965), are used. This greater influence is caused by the cohesion value being multiplied by the entire length of the failure surface. Therefore, the longer the failure surface the greater the influence of the cohesion value. In summary, for each combination of c' and ϕ', the factor of safety (FS) for each of the failed natural slopes was determined (Fig. 3).

The lognormal distribution was shown to fit these values of FS well according to the standard K-S Goodness-of-Fit Test for probability distributions. The K-S Test was not rejected at the 5% significance level for all combinations of design c' and ϕ' values and for the three types of slopes studied, i.e., natural, cut, and fill slopes. Even though the distribution type could be assumed to follow the lognormal distribution, estimates of the associated parameters, i.e., λ and ζ, would be precise only if a large number of failed cases have been used for the estimation. (λ and ζ are defined as the expected value and standard deviation of the natural logarithm of the factor of safety, respectively.) Since a limited number of landslide cases will be available, λ and ζ will be subject to uncertainties. The level of these uncertainties will depend on the number of failed cases available, and these uncertainties should be incorporated in determining the final distribution of the FS.

The first step in the proposed procedure involves the analysis of the uncertainty in λ and ζ for each combination of c' and ϕ'. According to Ang and Tang (1975), on the basis of the calculated FS values (namely FS_1 to FS_n) for the set of n failed slope cases, the joint distribution of λ and ζ follows a Gamma-Normal distribution as follows:

$$f''(\lambda,\zeta)=K\prod_{i=1}^{n}\left\{\frac{1}{\sqrt{2\pi}\zeta}\exp\left(-\frac{1}{2}\left[\frac{\ln FS_i-\lambda}{\zeta}\right]^2\right)\right\} \quad (1)$$

where K is a normalization constant. Based on the Theorem of Total Probabilities (Mood et al., 1987), incorporating the uncertainty in these parameters results in the final distribution of FS being obtained as:

$$f_{FS}(fs)=\int_0^{+\infty}\int_{-\infty}^{+\infty} f_{FS}(fs\,|\,\lambda,\zeta)f''(\lambda,\zeta)\,d\lambda\,d\zeta \quad (2)$$

where

$$f_{FS}(fs\,|\,\lambda,\zeta)=\frac{1}{\sqrt{2\pi}\zeta fs}\exp\left(-\frac{1}{2}\left[\frac{\ln fs-\lambda}{\zeta}\right]^2\right). \quad (3)$$

Figure 4 shows the histogram and probability density function curves with and without incorporation of the uncertainty associated with these parameters for the factor of safety for a natural slope with $c'=0.1$ kPa and $\phi'=30$ degrees (Fig. 2). Clearly, the incorporation of the uncertainty in the parameters λ and ζ increases the overall variability of the distribution as reflected by the increase in the width of the distribution. This also can be observed from the comparison of the mean and standard deviation of the distributions of FS, with and without incorporating the parameter uncertainties, shown in Table 2. The final distribution of FS can be assumed to be lognormal again with the updated mean and standard deviation as shown in the last column of Table 2. The width of this distribution conveys the uncertainty associated with the given analysis/design procedure. If an improved analysis/design procedure is used, the corresponding distribution is expected to be narrower. The model could generally be improved, for example, by the adoption of a three-dimensional limit equilibrium back-analysis (Stark and Eid, 1998) or a better assessment of the pore-water pressure condition at failure.

To determine the risk associated with a given design factor of safety, fs_d, the probability that the actual factor of safety will be below fs_d is used as the probability of failure. Hence,

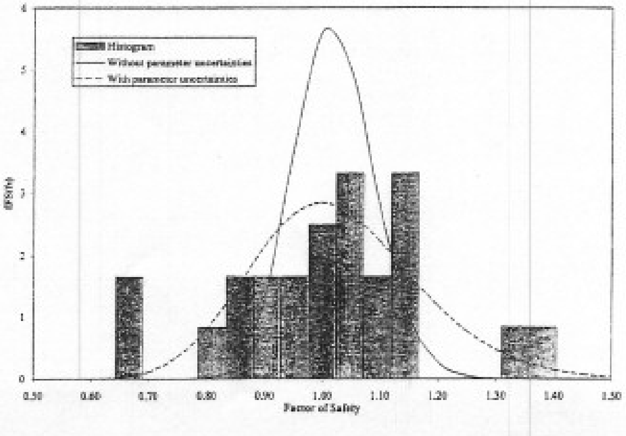


Fig. 4. Comparison of probability distributions of factor of safety after incorporation of parameter uncertainty for a natural slope with $c'=0.1$ kPa and $\phi'=30$ degrees

Table 2. Mean value and standard deviation of factor of safety for a natural slope

	Without incorporating parameter uncertainties	Incorporating parameter uncertainties
Mean factor of safety	1.00	1.00
Standard deviation of factor of safety	0.18	0.20

$$P_f=P(FS\le fs_d)=\int_0^{fs_d} f_{FS}(fs)dfs. \quad (4)$$

This can be easily computed using the standard normal distribution function as follows:

$$P_f=\Phi\left(\frac{\ln(fs_d)-\lambda}{\zeta}\right) \quad (5)$$

where λ and ζ can be determined from the mean and standard deviation of the factor of safety incorporating the parameter uncertainty. Consider a cut slope in the Orinda Formation with the combination of $c'=0.4$ kPa and $\phi'=35$ degrees, the value of P_f corresponding to the design FS of 1.5 is approximately 0.03, as shown in Fig. 5. The risk associated with other design factors of safety using different combinations of c' and ϕ' can be similarly estimated from Fig. 5. As expected, different combinations of effective stress cohesion and effective stress friction angle yield different probabilities of failure for the same factor of safety. Furthermore, there is an optimal combination of c' and ϕ' under which the probability of failure is the smallest, for a given design factor of safety. The combination of c' and ϕ' that yields the smallest probability of failure, for any given factor of safety for the cut slopes in the Orinda Formation, is 0.4 kPa and 35 degrees, respectively. Hence, that combination suggests an optimal choice of the design soil parameters for cut slopes in the Orinda Formation because it minimizes the risk associated with any design factor of safety.

In addition to obtaining the failure probability for a given factor of safety from the relationship shown in Fig. 5, the required factor of safety to achieve a desired risk level for the design of slopes can be estimated. For example, to achieve a probability of failure of 0.01 for future

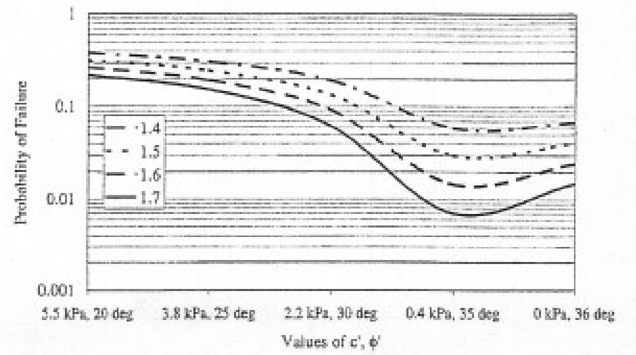


Fig. 5. Probability of failure for different factors of safety for cut slopes in the Orinda Formation

cut slopes in the Orinda Formation for the given soil shear strength parameters, $c'=0.4$ kPa and $\phi'=35$ degrees, the required design factor of safety would be about 1.65 (*see* Fig. 5). This factor of safety is higher than the typical 1.5 factor of safety for slopes, because it is based on a low probability of failure (0.01). Although factors of safety are assumed to correspond to a low probability of failure, the actual risk level is dependent on the uncertainty in the soil shear strength properties and other input parameters. As a result, a factor of safety of 1.5 can lead to different risk levels. In this example, a desired risk level of 0.01 results in a design factor of safety of 1.65. In summary, the proposed method allows the factor of safety to be related to the probability of failure. This allows the designer to assess the reliability associated with the design parameters (c', ϕ', and factor of safety). From a safety perspective, it is reasonable to maximize safety, i.e., factor of safety, and minimize the probability of failure.

The probability of failure for any given factor of safety is minimized at a certain combination of c' and ϕ'. This optimal combination is approximately constant for a given slope type. Figures 6 and 7 show the resulting relationships of probability of failure for the fill and natural slopes where 5 and 24 landslide cases are available, respectively. As expected, similar behavior is observed for these other types of slopes. However, the optimal combination of the effective stress shear strength parameters can be different for the two different slope types. The combination of c' and ϕ' for the case of fill slopes (Fig. 6) that results in the lowest probability of failure is approximately 1.8 kPa and 30 degrees, respectively, whereas for the cut slope case it is $c'=0.4$ kPa, and $\phi'=35$ degrees (Fig. 5). The optimal combination of c' and ϕ' for natural slopes is between the combinations of $c'=1.8$ kPa, $\phi'=25$ degrees and $c'=0.1$ kPa, $\phi'=30$ degrees (Fig. 7). For design purposes, it may be desirable to use the smaller value of c' and larger value of ϕ' because ϕ' is applied as a function of the effective normal stress along the failure surface whereas c' is assumed to be constant acting on the failure surface.

As expected, the fill and natural slopes exhibit a similar optimal combination of c' and ϕ' because the weathered materials of the Orinda Formation control the stability of both of these cases. The cut slope case probably exhibits a different and slightly higher combination of c' and ϕ' than the natural slopes because of the removal of the toe buttress and possible development of a progressive failure mechanism (Skempton, 1964). It is also possible that the drainage associated with the cut slopes was different and thus not able to dissipate some of the porewater pressures at the weathered/unweathered material interface, which resulted in a slightly higher combination of c' and ϕ'. This change in drainage could have been caused by a retaining structure being placed at the slope toe in some of the cases.

Again, the required design factor of safety for fill and natural slopes can be estimated from the relationships shown in Figs. 6 and 7 at the desired risk level. For example in Fig. 6, at a risk level of 0.01, the required factor of safety would be approximately 1.4. A lower design factor of safety is required for the case of fill slopes because of the relatively high values of factors of safety calculated for the five failed slopes.

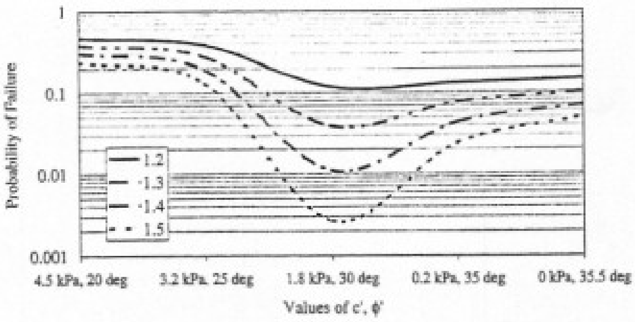


Fig. 6. Probability of failure for different factors of safety for fill slopes in the Orinda Formation

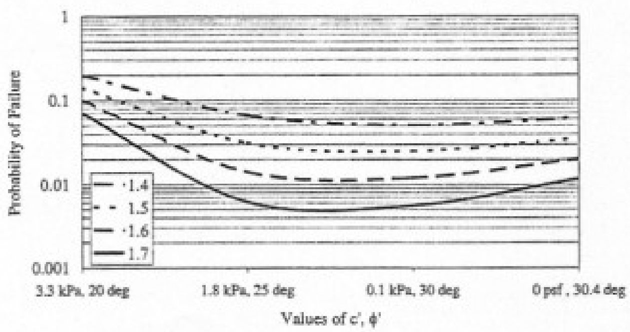


Fig. 7. Probability of failure for different factors of safety for natural slopes in the Orinda Formation

BENEFITS OF ADDITIONAL FAILED SLOPE CASE HISTORIES

The previous results are based on a limited number of failed slopes in the Orinda Formation. However, the results represent reasonable estimates of the implied risk after accounting for the uncertainties resulting from the limited data. However, perhaps increasing the amount of data by using additional slope failures would affect the analysis. Clearly with additional data, the uncertainty in the estimated parameters, and hence the width of the final distribution of *FS*, should be reduced. Accordingly, the risk associated with a given design factor of safety should also be reduced. In other words, a smaller design factor of safety may be adequate in achieving the same desired level of slope safety. The following paragraphs investigate the reduction in the design factor of safety that could be achieved with additional case histories.

For a required minimum risk level, say a probability of failure equal to 0.001; additional data could yield larger or smaller values of factor of safety. On the basis of the current set of data, the additional data is expected to yield values of mean and standard deviation that are comparable to the current ones; however, the number of cases available, n, will be larger. The effect of doubling the amount of data for each of the three slope types (namely 20, 10 and 48 for cut, fill and natural slopes, respectively) is shown in Table 3. The values in Table 3 were obtained by following the same procedure presented above with the appropriate value of n in Eq. (1). As expected, the results show a decrease in the design factor of safety with an increase in the number of available case histories. Therefore, the design factor of safety decreases with increasing n but the rate of decrease, does not equal the reciprocal of the square root of n. While the required design factor of safety to achieve a risk level of 0.001 does not change appreciably for the natural slopes (from 1.91 to 1.87), it changes significantly for the fill slopes (from 1.56 to 1.33) and for the cut slopes (from 1.95 to 1.78). This means that the amount of data for the natural slopes is sufficiently large that the benefit of additional case histories is not as great.

In fact, the maximum benefit of additional data could be studied by considering the case where infinite data were indeed available. The appropriate equations become:

$$f''(\lambda,\zeta)=K\sum_{i=1}^{\infty}\left\{\frac{1}{\sqrt{2\pi}\zeta}\exp\left(-\frac{1}{2}\left[\frac{\ln FS_i-\lambda}{\zeta}\right]^2\right)\right\} \quad (6)$$

where K is a normalization constant. In this case, the distribution of λ and ζ becomes very narrow around the modal values, and the uncertainty is only due to the variability in the factor of safety. The design factor of safety is obtained from:

$$fs_d=\exp\left[\Phi^{-1}(P_f)\zeta+\lambda\right]. \quad (7)$$

The corresponding design factors of safety are shown in the last column of Table 3. It can be seen that the required design factor of safety for natural slopes changes from 1.91 to 1.73 if an infinite amount of data is available. In summary, this analysis suggests that 25 to 30 case histories are needed to obtain a reasonable estimate of the design factor of safety for natural slopes in the Orinda Formation.

The design factors of safety in Table 3 correspond to a failure probability of 0.001. If the desired failure probability could be higher, then the design factors of safety could be reduced. Therefore, a factor of safety of 1.5 would have a probability of failure that is higher than 0.001 for all three slope types. The design factor of safety for the case of infinite data exceeds the usual design value of 1.5 for cut and natural slopes. The high values of factor of safety reflect uncertainties in the design/performance of these types of slopes. The use of higher factors of safety is usually used in design to compensate for uncertainties in the input parameters. For example, the U.S. Environmental Protection Agency (U.S EPA 1988) recommends different minimum factors of safety for different consequences of failure and different uncertainties in shear strength measurements (see Table 4) for landfill slopes. It can be seen that the minimum factor of safety exceeds 1.5 when there is large uncertainty in the shear strength measurements and significant environmental consequence of failure. However, the minimum factor of safety can be 1.25 if the uncertainty in the shear strength measurement is small and there is no significant environmental consequence of failure.

Table 3. Effect of additional data on the design factor of safety for different types of slope and a probability of failure of 0.001

Type of slope	Number of available cases	Design Factor of Safety		
		Available data	Twice the available data	Infinite data
Cut Slopes	10	1.95	1.78	1.67
Fill Slopes	5	1.56	1.33	1.26
Natural Slopes	24	1.91	1.87	1.73

Table 4. Recommended minimum values of factor of safety for slope stability analyses (from U.S EPA 1988)

Consequence of Slope Failure	Uncertainty of shear strength measurements	
	Small[1]	Large[2]
No imminent danger to human life or major environmental impact if slope fails	1.25 (1.2)*	1.5 (1.3)*
Imminent danger to human life or major environmental impact if slope fails	1.5 (1.3)*	2.0 or greater (1.7 or greater)*

Notes:
1. The uncertainty of the strength measurements is smallest when the soil conditions are uniform and high quality strength test data provide a consistent, complete, and logical picture of the strength characteristics.
2. The uncertainty of the strength measurements is greatest when the soil conditions are complex and when available strength data do not provide a consistent, complete, or logical picture of the strength characteristics.
* Factor of safety applies to seismic conditions.

SUMMARY AND CONCLUSION

Estimating soil shear strength parameters from laboratory tests introduces uncertainty into slope stability computations. Back analysis is a useful procedure for estimating the field or mobilized soil shear strength directly from slope failures. The advantages of back calculated shear strength include avoidance of soil disturbance and determination of soil shear strength that is representative of the soil mass over a large area. As a result, the back calculated shear strength reflects the influences of soil fabric, fissures, pre-existing shear surfaces, and long-term loading. Although back analysis is a useful procedure for estimating field values of soil shear strength, it cannot provide a unique combination of the Mohr-Coulomb shear strength parameters, c' and ϕ', for the soil involved in a landslide, nor can it be used without assumptions. A procedure is presented herein to determine the probability of failure for a given limit equilibrium slope stability analysis, factor of safety, and combination of c' and ϕ'. In addition, the design factor of safety can be estimated for a given risk level and combination of c' and ϕ' for the stability analysis presented. This risk analysis presented herein was developed using Bishop's (1955) modified slope stability method but it can be extended to other types of limit equilibrium slope stability method.

ACKNOWLEDGMENTS

This study is supported in part by a RGC Competitive Earmarked Research Grant 96/97 number HKUST722/96E. This study was also performed by the second author as a part of National Science Foundation Grant Number BCS-93-00043. 1 The support of this agency is gratefully acknowledged. The authors acknowledge the assistance of Morgan B. Finch, an undergraduate research assistant at the University of Illinois, in preparing the manuscript. The second author also acknowledges the support provided by the William J. and Elaine F. Hall Scholar Award.

REFERENCES

1) Ang, A. H.-S. and Tang, W. H. (1975): *Probability Concepts in Engineering Planning and Design-Volume I-Basic Principles*, John Wiley & Sons, Inc., New York, NY.
2) Bishop, A. W. (1955): "The use of the slip circle in the stability analysis of slopes." Géotechnique, Vol. 5, No. 1, pp. 7–17.
3) Duncan, J. M. (1992): "State-of-the-art: Static stability and deformation analysis." Proc. of Specialty Conf. STABILITY AND PERFORMANCE OF SLOPES AND EMBANKMENTS-II, ASCE, Berkeley, CA, June, Vol. 1, pp. 222–266.
4) Duncan, J. M. (1996): "Soil slope stability analysis." LANDSLIDES: Investigation and Mitigation, Transportation Research Board Special Report 247, edited by A. Keith Turner and Robert L. Schuster, National Research Council, National Academy of Sciences, Washington, D.C., pp. 337–371.
5) Duncan, J. M. and Stark, T. D. (1992): "Soil strengths from back-analysis of slope failures." Proc. of Specialty Conf. STABILITY AND PERFORMANCE OF SLOPES AND EMBANKMENTS-II, ASCE, Berkeley, CA, June, Vol. 1, pp. 890–904.
6) Duncan, J. M. and Wright, S. G. (1980): "The accuracy of equilibrium methods of slope stability," Proc. of 3rd Int. Symp. On Landslides, New Delhi, pp. 247–254.
7) Fredlund, D. G. and Krahn, J. (1977): "Comparison of slope stability methods of analysis." Can. Geotech. J., Vol. 14, No. 3, pp. 429–439.
8) Janbu, N. (1968): "Slope stability computations." Soil Mechanics and Foundation Engineering Report, The Technical Univ. of Norway, Trondheim.
9) Ladd, C. C., Foote, R., Ishihara, K., Schlosser, F., and Poulos, H. G. (1977): "Stress-deformation and strength characteristics." State-of-the-Art Report, Proc. of 9th Int. Conf. on SMFE, Tokyo, Vol. 2, pp. 421–494.
10) Leshchinsky, D. (1990): "Slope stability analysis: Generalized approach." ASCE, J. of Geotech. Engrg., Vol. 116, No. 5, pp. 851–867.
11) Mitchell, J. K. (1993): Fundamentals of soil behavior, Géotechnique, John Wiley & Sons, Inc., New York, 436 pp.
12) Mood, A. M., Graybill, F. A., and Boes, D. C. (1987): Introduction to the Theory of Statistics, McGraw-Hill, New York, NY, p. 356.
13) Morgenstern, N. R. and Price, V. W. (1965): "The analysis of the stability of general slip surfaces," Géotechnique, Vol. 15, No. 1, pp. 79–93.
14) Radbruch, D. H. and Weiler, L. M. (1963): "Preliminary report on landslides in a part of the orinda formation, Contra costa county, California." U.S. Geological Survey Open File Report, 35 p.
15) Skempton, A. W. (1964): "Long term stability of clay slopes." Géotechnique. Vol. 35, No. 1, pp. 3–18.
16) Spencer, E. (1967): "A method of analysis of the stability of embankments assuming parallel interslice forces," Géotechnique, Vol. 17, No. 1, pp. 11–26.
17) Stark, T. D. and Eid, H. T. (1998): "Performance of three-dimensional slope stability methods in practice," J. of Geotech. and Geoenvir. Engrg., ASCE, Vol. 124, No. 11, November, 1998, pp. 1049–1060.
18) U.S. Environmental Protection Agency (1988): "Guide to technical resources for the design of land disposal facilities." EPA Report EPA/625/6-88/o18, Office of Research and Development, Cincinnati, Ohio.
19) Wright, S. G., Kulhawy, F. H., and Duncan, J. M. (1973): "Accuracy of equilibrium slope stability analysis." ASCE, J. of Soil Mechanics and Foundations Division, Vol. 99, No. 10, pp. 783–791.