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Effective Stress Hyperbolic Stress-Strain Parameters for Clay

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ABSTRACT: This paper presents a procedure for estimating the effective stress hyperbolic stress-strain parameters for normally consolidated clays from the results of consolidation and direct shear tests. The procedure for calculating Young's modulus and the modulus number includes: (1) estimating the failure ratio from the shear stress–horizontal displacement curve obtained from a direct shear test; (2) using a tangent modulus at the end of each normally consolidated load increment in the consolidation test and the void ratio at the beginning of each load increment to calculate Young's modulus; and (3) multiplying the resulting modulus number by 1.9 to obtain a reasonable estimate of the isotropically consolidated-drained triaxial (CID) modulus number. The modulus exponent was approximately unity for both the consolidation and CID triaxial tests. A procedure for estimating the bulk modulus number, the bulk modulus exponent, and the unload-reload modulus number is also presented.

KEY WORDS: clays, effective stress, finite element, elasticity modulus, deformation modulus

Nomenclature

- a_v Coefficient of compressibility
- B Bulk modulus
- c' Effective cohesion
- e_0 Initial void ratio
- E_i Initial tangent modulus
- E_t Tangent modulus
- E_{ur} Unload-reload modulus
- K Modulus number
- K_b Bulk modulus number
- K_0 At-rest earth pressure coefficient
- K_0^u Unloading earth pressure coefficient
- K_{ur} Unload-reload modulus number
- m Bulk modulus exponent
- m_v Coefficient of volume change
- n Modulus exponent
- p_a Atmospheric pressure (101.4 kPa)
- R_f Failure ratio
- $\Delta\varepsilon$ Change in vertical strain
- $\Delta\sigma_1'$ Change in effective major principal stress
- Δx Horizontal displacement
- ε Axial strain
- σ_v' Vertical effective stress
- σ_3' Effective confining stress
- $(\sigma_1' - \sigma_3')$ Effective deviator stress
- $(\sigma_1' - \sigma_3')_f$ Effective deviator stress at failure
- $(\sigma_1' - \sigma_3')_{ult}$ Ultimate deviator stress
- τ Shear stress
- τ_f Shear stress at failure
- τ_{ult} Ultimate shear stress
- ϕ' Effective stress friction angle

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Introduction

The hyperbolic stress-strain relationships were developed by Duncan and Chang (1970) for use in nonlinear finite element analyses of soil stresses and deformations. The hyperbolic stress-strain model is very popular because it can be used for both effective stress and total stress analyses, the hyperbolic parameters can be easily determined from conventional triaxial compression tests, and an extensive database of total and effective stress hyperbolic parameters has been developed by Duncan et al. (1980). The model has been successfully applied to embankment dams, open excavations, braced excavations, and a variety of soil-structure interaction problems (Chang 1969; Clough and Duncan 1969; Duncan et al. 1990; Mana and Clough 1981; Seed and Duncan 1986).

Duncan et al. (1980) provide an extensive derivation of the hyperbolic model and a detailed procedure for determining the values of the hyperbolic stress-strain parameters from conventional triaxial tests. As a result, only the major features of the model will be described in this introduction in order to define the various hyperbolic stress-strain parameters.

The hyperbolic model represents the nonlinear stress-strain curve of soils using a hyperbola as shown in Fig. 1. It can be seen that transforming the hyperbolic equation results in a linear relationship between $\varepsilon/(\sigma_1' - \sigma_3')$ and ε, where ε is the axial strain and $(\sigma_1' - \sigma_3')$ is the effective deviator stress. The stress-dependent stress-strain behavior of soil is represented by varying the initial tangent modulus, E_i, and the ultimate deviator stress, $(\sigma_1' - \sigma_3')_{ult}$, with the effective confining pressure, σ_3'. It can be seen from Fig. 1 that the ultimate deviator stress is the asymptotic value of the deviator stress and is related to the compressive strength of the soil. The variation of the initial tangent modulus with confining pressure is represented by an empirical equation proposed by Janbu (1963)

$$E_i = Kp_a\left(\frac{\sigma_3'}{p_a}\right)^n \quad (1)$$

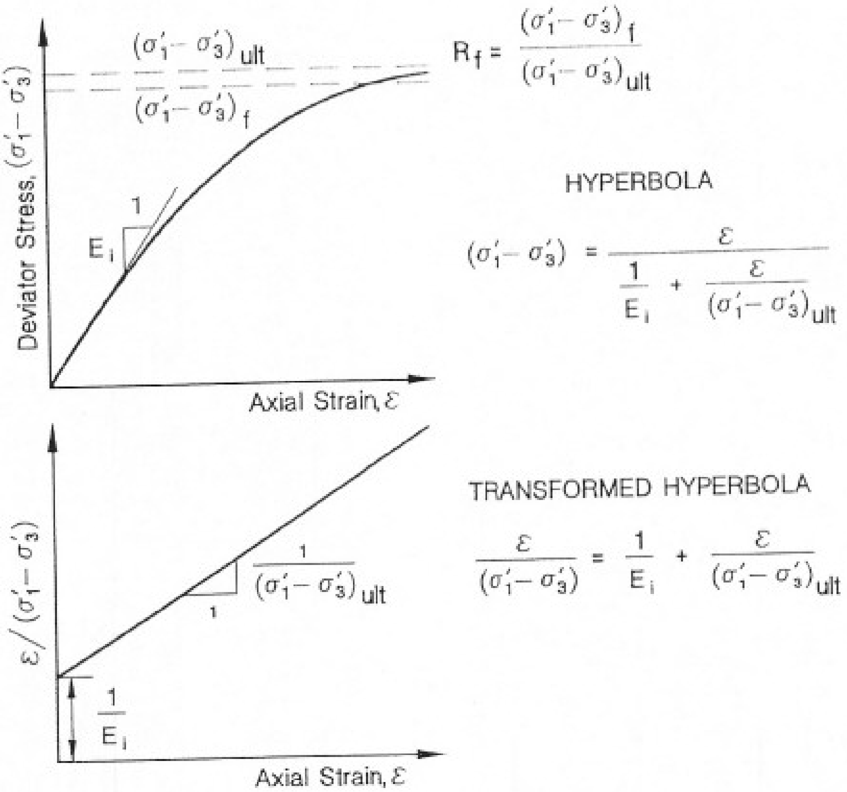

FIG. 1—*Hyperbolic representation of a stress-strain curve (Duncan et al. 1980).*

where

K = modulus number,
n = modulus exponent, and
p_a = atmospheric pressure in the same units as σ_3' and E_i.

The variation of E_i with σ_3' is linear when the logarithm of (E_i/p_a) and (σ_3'/p_a) are plotted against each other. The modulus number equals (E_i/p_a) at a value of (σ_3'/p_a) equal to one and n is the slope of the resulting line.

The variation of ultimate deviator stress with σ_3' is accounted for by relating $(\sigma_1' - \sigma_3')_{ult}$ to the stress difference at failure, $(\sigma_1' - \sigma_3')_f$, and using the Mohr-Coulomb strength equation to relate $(\sigma_1' - \sigma_3')_f$ to σ_3'. The criteria used to define $(\sigma_1' - \sigma_3')_f$ is usually the maximum deviator stress. However, the criteria which results in the best approximation of the actual stress-strain curve should be used. The values of $(\sigma_1' - \sigma_3')_{ult}$ and $(\sigma_1' - \sigma_3')_f$ are related by

$$(\sigma_1' - \sigma_3')_f = R_f(\sigma_1' - \sigma_3')_{ult} \tag{2}$$

in which R_f is the failure ratio as shown in Fig. 1. The value of R_f is always less than or equal to 1.0 and varies from 0.5 to 0.9 for most soils. The variation of $(\sigma_1' - \sigma_3')_f$ with σ_3' can be expressed as follows using the Mohr-Coulomb strength equation

$$(\sigma_1' - \sigma_3')_f = \frac{2c' \cos\phi' + 2\sigma_3' \sin\phi'}{1 - \sin\phi'} \tag{3}$$

in which c' and ϕ' are the effective stress Mohr-Coulomb cohesion intercept and friction angle, respectively.

By differentiating the equation of a hyperbola shown in Fig. 1 with respect to the axial strain and substituting the expression into Eqs 1, 2, and 3, an expression for the tangent modulus, E_t, can be obtained

$$E_t = \left[1 - \frac{R_f(1 - \sin\phi')(\sigma_1' - \sigma_3')}{2c' \cos\phi' + 2\sigma_3' \sin\phi'}\right]^2 Kp_a\left(\frac{\sigma_3'}{p_a}\right)^n \tag{4}$$

This equation can be used to calculate the value of E_t for any stress condition if the hyperbolic parameters K, n, and R_f and the Mohr-Coulomb shear strength parameters, c' and ϕ', are known.

The hyperbolic stress-strain model accounts for the nonlinear volume change behavior of soils by assuming that the bulk modulus is independent of stress level, $(\sigma_1' - \sigma_3')$, and that it varies with confining pressure. The variation of bulk modulus, B, with confining pressure is approximated by the following equation

$$B = K_b p_a\left(\frac{\sigma_3'}{p_a}\right)^m \tag{5}$$

where

K_b = the bulk modulus number, and
m = the bulk modulus exponent.

The variation of B is linear when the logarithm of (B/p_a) and (σ_3'/p_a) are plotted against each other. The bulk modulus number equals (B/p_a) at a value of (σ_3'/p_a) equal to one and m is the slope of the resulting line.

Determining Effective Stress Hyperbolic Stress-Strain Parameters for Clays

Due to the low permeability of cohesive soils, isotropically consolidated-drained (CID) triaxial tests are rarely performed on clayey specimens. As a result, Clough and Duncan (1969) developed a procedure for estimating the effective stress hyperbolic stress-strain parameters for clays using the results of consolidation and direct shear tests. The direct shear tests are used to determine the effective stress cohesion, c', and friction angle, ϕ', and the consolidation test yields a stress-strain curve for the primary loading (loading beyond the preconsolidation pressure) condition. Clough and Duncan (1969) derived the following expression for determining the initial tangent modulus, E_i, in terms of E_t for any load increment in a consolidation test

$$E_i = \frac{E_t}{\left[1 - \frac{R_f(1 - K_0)}{K_0[\tan^2(45 + \phi'/2) - 1]}\right]^2} \quad (6)$$

where K_0 equals the at-rest earth pressure coefficient. The major assumption incorporated into Eq 6 is that c' equals zero, which is a typical value for normally consolidated soils. Values of (E_i/p_a) and (σ_3'/p_a) for different load increments are plotted on logarithmic scales to obtain a straight line from which K and n can be determined. The average value of σ_3' during each load increment is obtained by multiplying the average vertical stress during the increment by K_0. The tangent modulus can be derived from a consolidation curve using the following equation developed by Chang (1969)

$$E_t = \frac{1 + e_0}{a_v}\left[1 - \frac{2K_0^2}{(1 + K_0)}\right] \quad (7)$$

where

e_0 = the void ratio at the beginning of a load increment, and
a_v = the coefficient of compressibility during primary loading.

The coefficient of compressibility is assumed to be a tangent

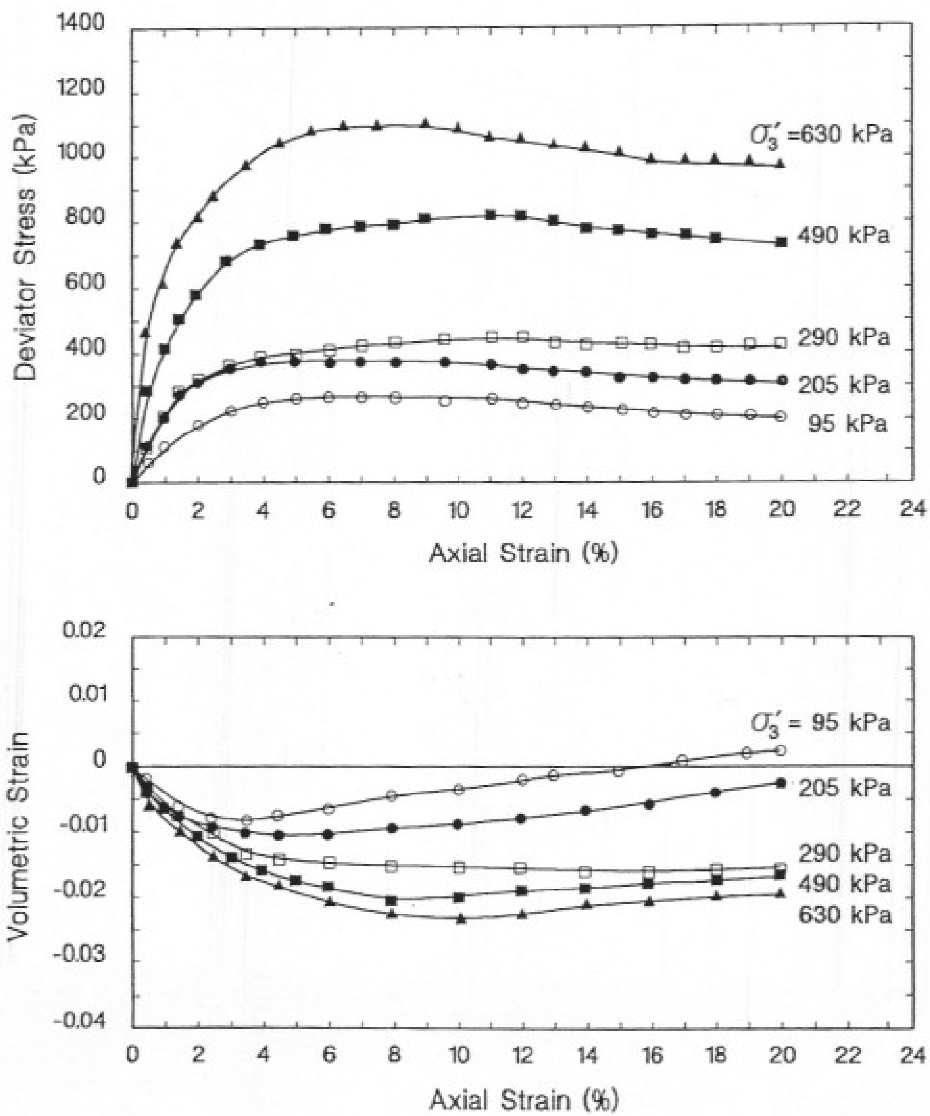

FIG. 2—*Stress-strain and volumetric strain curves from CID triaxial tests.*

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modulus at the end of a load increment when the oedometer test results are plotted arithmetically.

The bulk modulus is defined as the change in mean stress divided by the change in volumetric strain and can be approximated from a consolidation test using the following equation

$$B = \frac{\Delta\sigma_1'(1 + 2K_0)}{3\Delta\varepsilon} \quad (8)$$

where

$\Delta\sigma_1'$ = the change in the effective major principal stress, and
$\Delta\varepsilon$ = the corresponding change in vertical strain.

The variation of (E_i/p_a) and (σ_3'/p_a) is again linear when plotted on logarithmic scales and the values of K_b and m can be easily determined. The average value of σ_3' during each load increment is calculated using the average vertical effective stress during the increment and K_0.

The procedure for determining the effective stress hyperbolic stress-strain parameters for clays from the results of consolidation and direct shear tests has been widely used. However, the accuracy of the procedure has not been verified using the results of CID triaxial tests. This paper provides a comparison of the hyperbolic stress-strain parameters determined from CID triaxial tests and consolidation and direct shear tests. Based on this comparison, a new procedure for calculating the hyperbolic parameters from consolidation and direct shear tests is presented.

Hyperbolic Parameters from Isotropically Consolidated-Drained Triaxial Tests

Five CID triaxial tests were performed on the clayey slopewash from the foundation of San Luis Dam. San Luis Dam is located in the Central Valley of California approximately 100 miles southeast of San Francisco. The slopewash is a medium to high plasticity clay with liquid and plastic limits equal to approximately 40 and 20, respectively. The slopewash has an overconsolidation ratio ranging from 1.0 to 1.5 and is classified as a CL according to the Unified Soil Classification system. Extensive consolidation and direct shear tests were conducted previously by the primary author (Stark 1987) using undisturbed block samples provided by the U.S. Bureau of Reclamation. All the slopewash specimens used in the CID triaxial, consolidation, and direct shear tests were obtained from the same block sample which was obtained from the upstream foundation of San Luis Dam.

An axial strain rate of 0.0027% per minute was used in the CID triaxial tests to ensure drained conditions during loading. The strain rate was determined using the methodology presented by Gibson and Henkel (1954) and a coefficient of consolidation equal to 0.11 cm²/min. The CID triaxial tests were continued until the axial strain reached approximately 20%, which took about five to six days. Prior to loading, the triaxial specimens were back-pressured until a "B" value, which equals the change in pore pressure divided by the change in confining stress, of 0.97 or higher was obtained. It usually required seven to ten days for the specimens to obtain the desired "B" value. The specimens were then consolidated using confining pressures, which ensured the specimens would exhibit a normally consolidated behavior. The resulting stress-strain and volume change curves from the five CID triaxial tests are shown in Fig. 2. The effective stress hyperbolic and Mohr-Coulomb strength parameters obtained from these tests using the procedure outlined by Duncan et al. (1980) are shown in Table 1. The modulus number and bulk modulus number were obtained from Figs. 3 and 4, respectively.

Hyperbolic Parameters from Consolidation and Direct Shear Tests

Using Eqs 6 and 7 and the results of three consolidation tests (Fig. 5) and six direct shear tests on undisturbed slopewash specimens, the hyperbolic stress-strain and Mohr-Coulomb strength parameters shown in Table 2 were obtained. Figures 3 and 4 were also used to determine the hyperbolic stiffness and volume change parameters shown in Table 2.

It can be seen from Tables 1 and 2 that the modulus number obtained from the consolidation and direct shear test results significantly underestimated the triaxial modulus number. The difference in the modulus numbers is probably due to the different boundary conditions and stress paths inherent in these tests. However, the value of the modulus exponent was approximately unity for both cases. It can be shown using critical state soil mechanics that the modulus exponent should be unity for soils undergoing primary loading, which is the case for the slopewash. The following sections illustrate the influence of various factors on the modulus number derived from consolidation and direct shear test results.

Effect of Friction Angle, K_0 and R_f on the Hyperbolic Stiffness Parameters

A parametric study revealed that the influence of the friction angle on E_i (Eq 6) is very small. The effective friction angle for the slopewash was measured to be 25 and 28 in direct shear and CID triaxial tests, respectively. The values of K obtained from Eq 6 for friction angles of 25 and 28 were 80 and 75, respectively. The friction angle was also found to have no effect on the modulus exponent. Therefore, it was concluded that using the friction angle obtained from direct shear tests would not significantly affect the estimates of K or n. In fact, the direct shear friction angle increased the value of K, which results in better agreement with the triaxial value of K.

The parametric study also revealed that variations in K_0 had a significant effect on K and a negligible effect on n. This is due to the importance of K_0 in Eqs 6 and 7, which are used to calculate E_i and E_r, respectively. As a result, the value of K_0 should be chosen carefully. The values of K_0 used in this investigation were obtained from data presented by Brooker and Ireland (1965).

The value of R_f reported in Table 2 was calculated using the following equation

$$R_f = \frac{\tau_f}{\tau_{ult}} \quad (9)$$

where

τ_f = the shear stress at failure, and
τ_{ult} = the ultimate shear stress.

Both of these parameters should be determined from the shear stress measured during the first travel of the shear box in a direct shear test. The ultimate shear stress is the asymptotic value of the shear stress and is calculated using the shear stress, τ, and

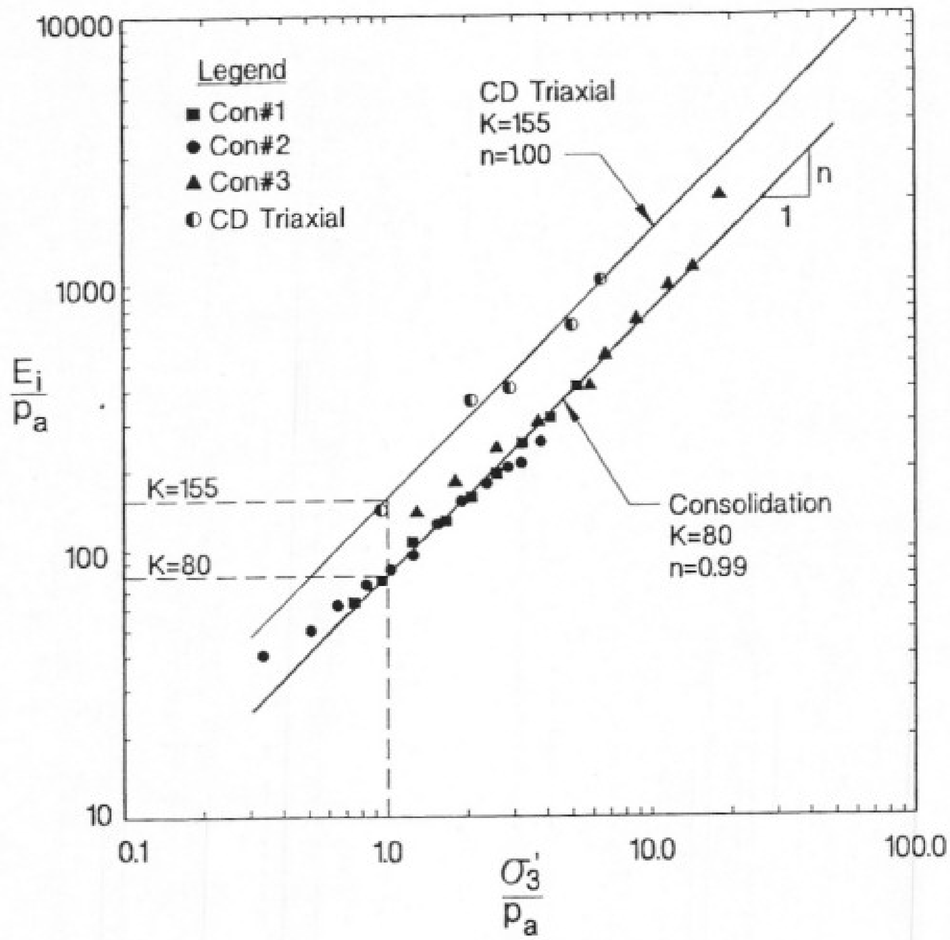

FIG. 3—*Hyperbolic stiffness parameters from CID triaxial and consolidation tests.*

horizontal displacement, Δx, at 70 and 95% of τ_f as shown in the following equation

$$\tau_{ult} = \frac{\Delta x_{95\%} - \Delta x_{70\%}}{\frac{\Delta x_{95\%}}{\tau_{95\%}} - \frac{\Delta x_{70\%}}{\tau_{70\%}}} \quad (10)$$

Therefore Eqs 9 and 10 provide a means for estimating R_f from direct shear test results instead of assuming a value of 0.85 as suggested by Clough and Duncan (1969).

This procedure for estimating R_f is similar to that outlined by Duncan et al. (1980) for determining R_f from triaxial tests except the horizontal displacement and shear stress are used instead of axial strain and deviator stress. Initially, efforts were made to normalize the horizontal displacement using the height or length of the direct shear specimen. This might have provided a better correspondence between a triaxial stress-strain curve and the shear stress-displacement curve from direct shear tests. However, if the displacements shown in Eq 10 are divided by 10.2 cm, the length of the direct shear specimen, the value of τ_{ult} does not change. Therefore, the horizontal displacement at 70 and 95% of the peak shear stress is used in Eq 10.

It can be seen from Tables 1 and 2 that the average value of R_f from the direct shear tests, 0.98, was about 30% higher than the average value of R_f from the CID triaxial tests, 0.72. It can be seen from Fig. 6 that increasing R_f from 0.72 to 0.98 increased the value of K from 45 to 80 and did not affect the value of n. Thus a higher value of R_f appears to provide a better estimate of the CID triaxial modulus number. Since the direct shear test results provide a value of R_f that is higher than the value of 0.85 proposed by Clough and Duncan (1969), it is recommended that R_f be determined directly from direct shear test results.

Effect of E_t on the Hyperbolic Stiffness Parameters

Values of tangent modulus were calculated using various values of e_0 and a_v from the consolidation test results and Eq 7. In accordance with Clough and Duncan's (1969) procedure, the values of K and n shown in Table 2 and Fig. 3 were obtained using the void ratio at the beginning of each load increment and a tangent modulus at the end of each load increment. The value of σ_3' used in the calculations was the average value of σ_3' over the load increment. The effective confining pressure was estimated using an appropriate value of K_0.

In an effort to improve the estimate of K from the consolidation test data, different values of e_0 and tangent and secant moduli were used in the calculations. The tangent modulus was always taken at the end of the load increment, and the values of secant modulus were determined over a particular load increment in the consolidation test. Figure 7 illustrates the difference between the tangent and secant moduli used in the calculations for consolidation test CON#3. In all cases the value of

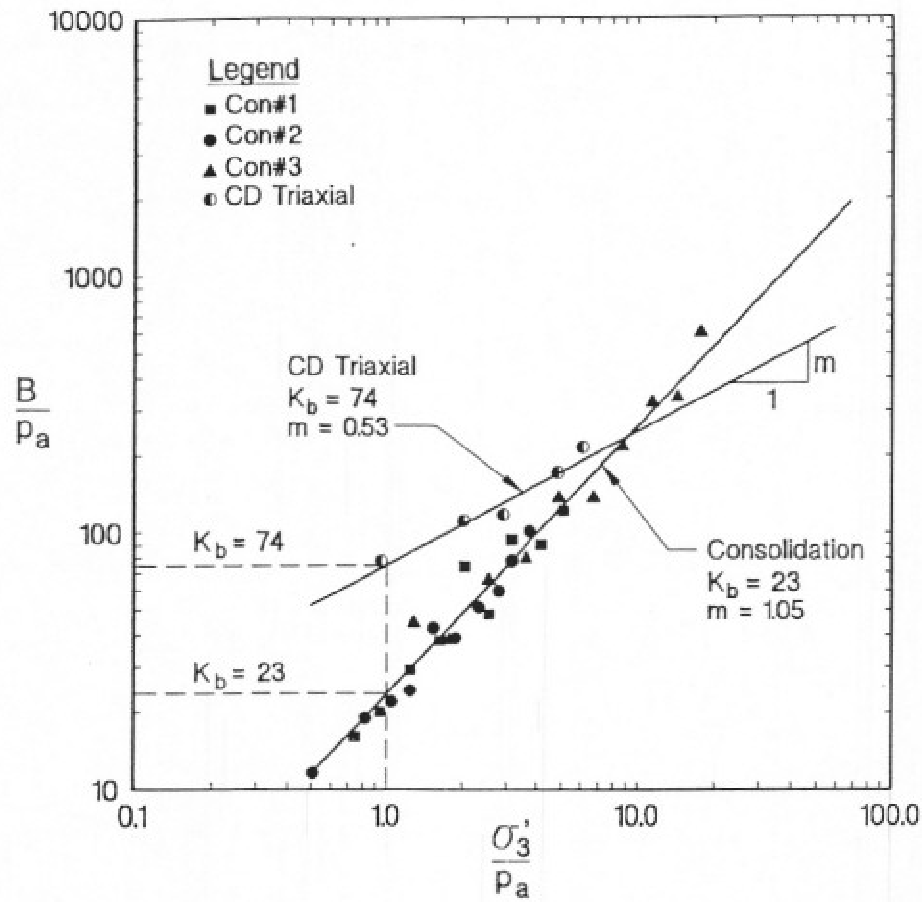

FIG. 4—*Bulk modulus parameters from CID triaxial and consolidation tests.*

TABLE 1—*Effective stress hyperbolic and Mohr-coulomb parameters from CID triaxial tests.*

Parameter	Value
Stress range	50 to 600 kPa
K	155
n	1.00
R_f	0.72
K_b	74
m	0.53
c'	0
ϕ'	28

σ_3' was the average value over the load increment. It can be seen from Table 3 that a tangent modulus consistently yielded a higher value of K than a secant modulus. For both moduli, the effect of varying the void ratio from the beginning to the end of the load increment did not significantly affect the value of K. The value of 155 shown in Table 3 is the value of K determined from the CID triaxial test results. Therefore, the consolidation test underestimated the triaxial modulus number by a factor of 1.6 to 2.1 depending on the void ratio and the type of modulus used.

To investigate the effect of stress level on the secant modulus, each load increment in the consolidation test was divided into four subincrements and the secant modulus was calculated for each subincrement. The resulting value of K is shown at the bottom of Table 3 and is approximately the same as the previous values. Therefore, it was concluded that the stress level did not significantly affect the secant modulus. For all the combinations of void ratio and modulus considered in Table 3, the modulus exponent was found to be 0.99.

Since the coefficient of volume change, m_v, is defined as

$$m_v = \frac{a_v}{1 + e_0} \tag{11}$$

where e_0 is the initial void ratio, i.e., the void ratio at the start of the consolidation test was also used. The initial void ratio is substantially higher than the void ratio at the beginning or end of a particular load increment during a consolidation test. As a result, the initial void ratio provided a slightly better estimate of K (Table 3) for both the tangent and secant moduli.

The secant values of E_i were easier to calculate than the tangent values, but they showed considerably more scatter. As a result, more data were required to clearly define the linear relationship between E_i and σ_3' when a secant modulus was used. The tangent modulus was a little more subjective than a secant modulus, but it consistently gave a better estimate of K.

Therefore, it is recommended that the value of E_i (Eq 6) be estimated using a tangent modulus at the end of the load increment and the void ratio at the beginning of the increment. The resulting modulus number should then be multiplied by 1.9 to

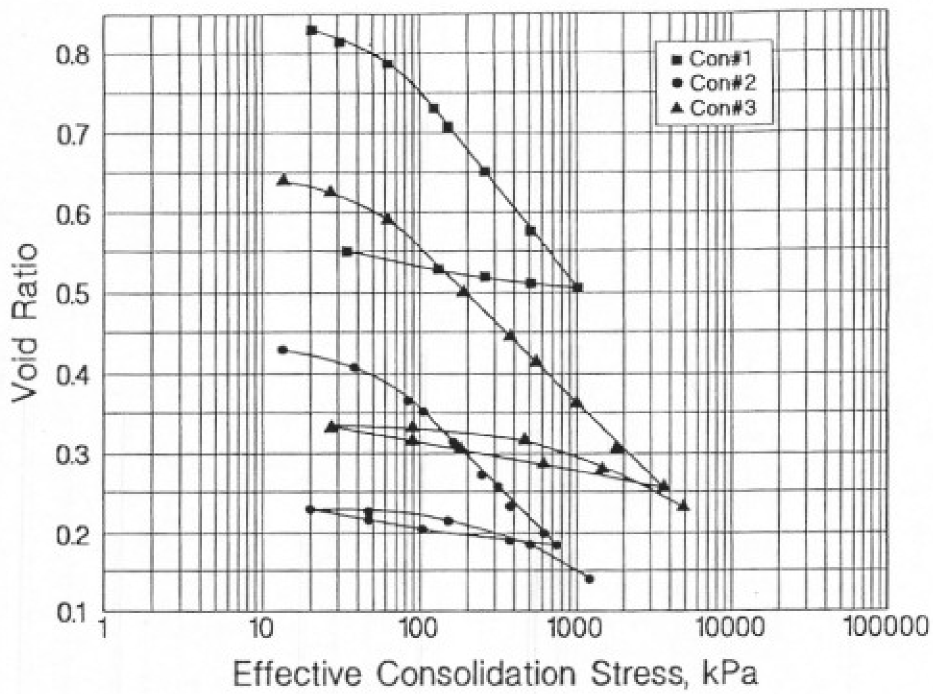

FIG. 5—*Void ratio-effective stress curves from one-dimensional consolidation tests.*

TABLE 2—*Effective stress hyperbolic and Mohr-coulomb parameters from consolidation and direct shear tests.*

Parameter	Value
Stress range	100 to 1000 kPa
K	80
n	0.99
R_f	0.98
K_b	23
m	1.05
c'	0
ϕ'	25

account for the differences between the boundary conditions and the stress paths in the consolidation and CID triaxial tests. The modulus exponent does not require adjustment if the soil is undergoing primary loading.

Determining the Bulk Modulus Number and Exponent

The values of bulk modulus were obtained using Eq 8 and the consolidation test results. It can be seen from Fig. 4 that the estimate of K_b and m from the consolidation test data were also in poor agreement with the CID triaxial values. The value of σ_3' used in Fig. 4 is the average value of σ_3' over a particular load increment. The major differences between consolidation and CID triaxial tests are: (1) the value of lateral stress increases with each load increment in the consolidation test and remains approximately constant in the triaxial test; and (2) the volumetric strain in the consolidation test is equal to the axial strain, whereas the volumetric strain in the triaxial test is the sum of the directional strains. These fundamental differences in the boundary conditions and stress paths in these two tests are believed to contribute significantly to the differences observed in K_b and m in Fig. 4. To obtain a reasonable estimate of the bulk modulus parameters determined from CID triaxial tests, it is recommended that the values of K_b and m determined from direct shear and consolidation tests be multiplied by 3.2 and 0.5, respectively.

Determining the Unload and Reload Modulus

In the hyperbolic stress-strain relationships, the same value of unload-reload modulus, E_{ur}, is used for both the unloading and reloading conditions. The value of E_{ur} is related to the confining pressure by an equation of the same form as Eq 1

$$E_{ur} = K_{ur}p_a\left(\frac{\sigma_3'}{p_a}\right)^n \tag{12}$$

where K_{ur} is the unload-reload modulus number. The value of the exponent n is assumed to be equal to the primary loading exponent. The value of K_{ur} is determined from a single unload-reload curve in a triaxial test. The best fit straight line is drawn from the unload point to the reload point as shown in Fig. 8. The corresponding value of E_{ur} (slope of the best fit line) is then determined. The value of K_{ur} is calculated using E_{ur}, the value of the confining pressure during unloading, the modulus exponent for primary loading, and Eq 12.

In the CID triaxial test conducted at a confining pressure of 205 kPa, the clayey slopewash was unloaded at a stress level of 0.52 and subsequently reloaded to evaluate K_{ur}. The stress level during primary loading in a consolidation test is determined using the following equation

$$\frac{(\sigma_1' - \sigma_3')}{(\sigma_1' - \sigma_3')_f} = \frac{(1 - K_0)}{K_0\,(\tan^2(45° + \phi'/2) - 1) + 2c'\tan(45° + \phi'/2))} \tag{13}$$

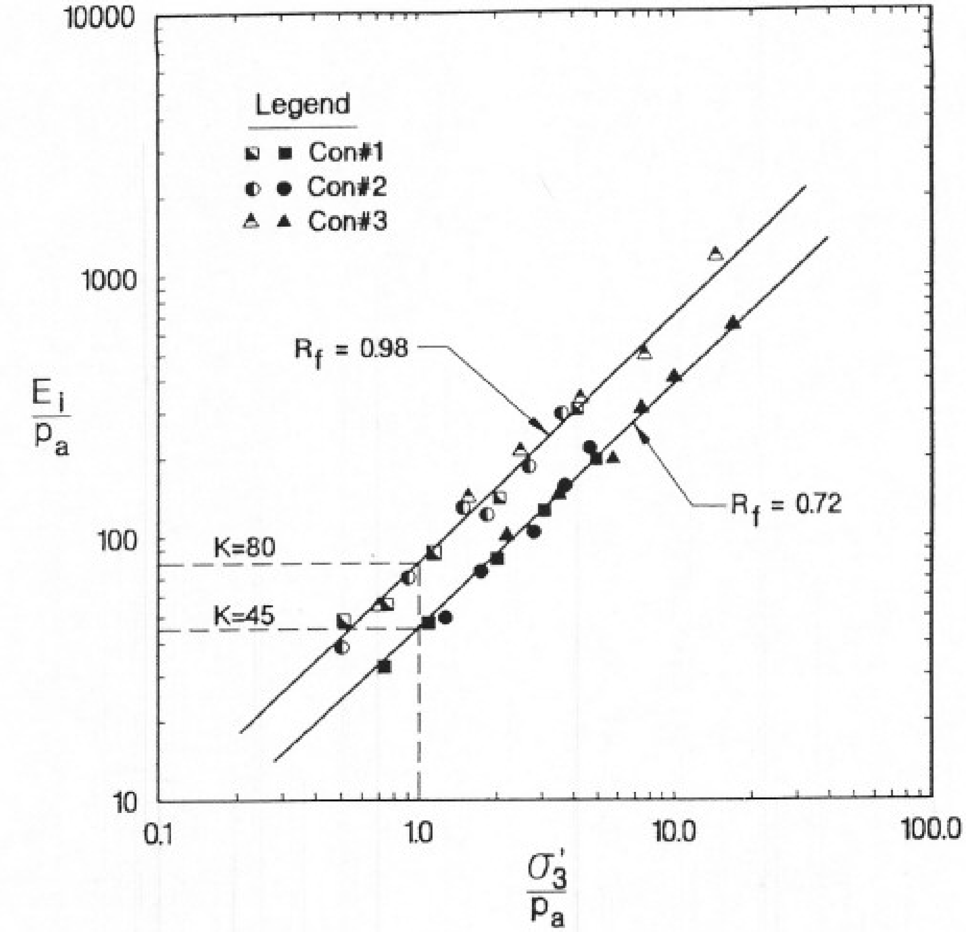

FIG. 6—*Effect of* R_f *on hyperbolic stiffness parameters determined from consolidation and direct shear tests.*

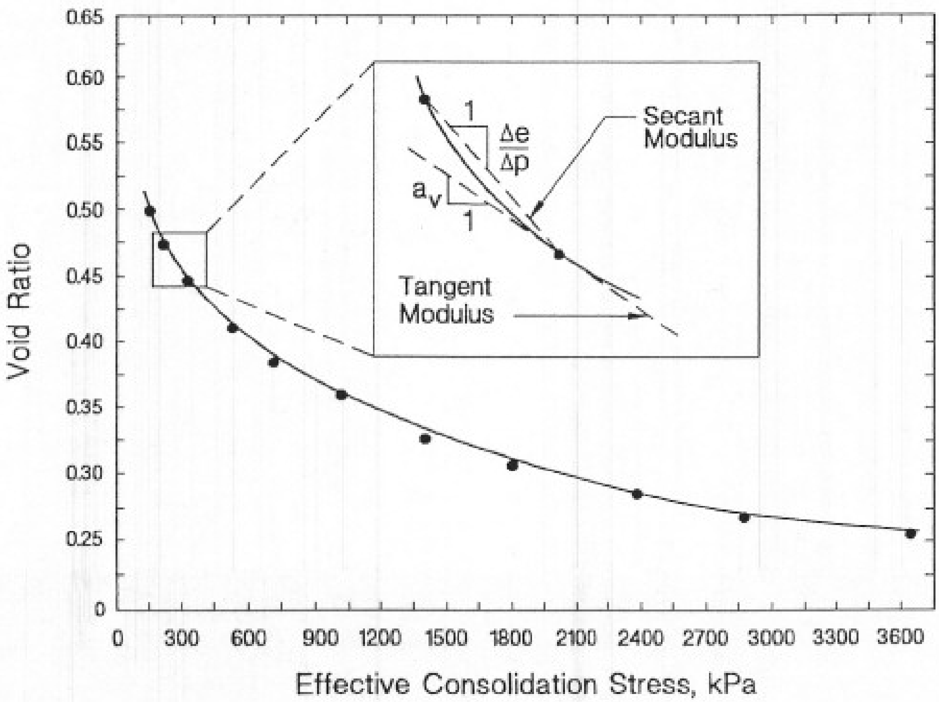

FIG. 7—*Secant and tangent moduli for consolidation test, CON#3.*

A stress level of 0.56 was calculated for the slopewash using Eq 13, and as a result the triaxial specimen was unloaded at a stress level of 0.52 for comparison purposes.

The stress-strain and volume change curves from the unload-reload CID triaxial test are shown in Fig. 8. The modulus number and the unload-reload modulus number for this test were calculated to be 155 and 285, respectively. This corresponds to a ratio of K_{ur}/K of 1.85. This is in good agreement with data presented by Duncan et al. (1980), which concludes that K_{ur}/K varies from about 1.2 for stiff soils up to 3 for soft soils.

Clough and Duncan (1969) also proposed a procedure for estimating K_{ur} from the rebound curve of a consolidation test. The unload-reload modulus is estimated using the following equation

$$E_{ur} = \frac{1 + e_0}{a_v}\left[1 - \frac{2(K_0^{\Delta})^2}{(1 + K_0^{\Delta})}\right] \quad (14)$$

where

e_0 = the initial void ratio, and
K_0^{Δ} = an incremental coefficient of lateral earth pressure during unloading.

The value of K_0^{Δ} was derived from the data presented by Brooker and Ireland (1965) and is the slope of the tangent to the unloading curve, whereas the value of K_0 is the slope of the secant line to

TABLE 3—*Effect of void ratio and modulus on the modulus number and exponent from consolidation tests.*

Void Ratio	Modulus	K	$155/K^a$
Beginning of load increment (Table 2)	Tangent	80	1.9
End of load increment	Tangent	79	2.0
Initial	Tangent	94	1.6
Beginning of load increment	Secant	74	2.1
End of load increment	Secant	74	2.1
Initial	Secant	83	1.9
Beginning of load increment	Secant[b]	72	2.2

Note: n = 0.99 for all seven procedures.
[a] K = 155 from CID triaxial tests.
[b] Secant over one fourth of actual load increment.

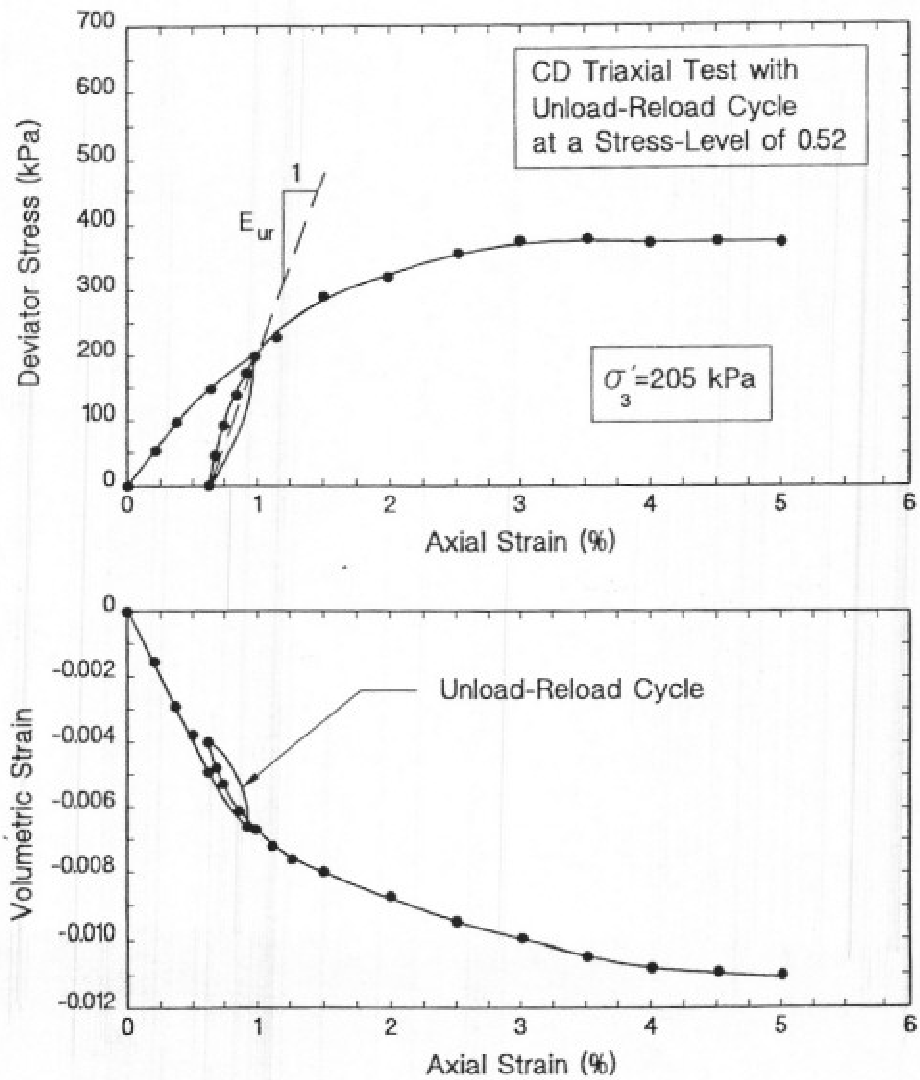

FIG. 8—*Stress-strain and volumetric strain curves from unload-reload CID triaxial test.*

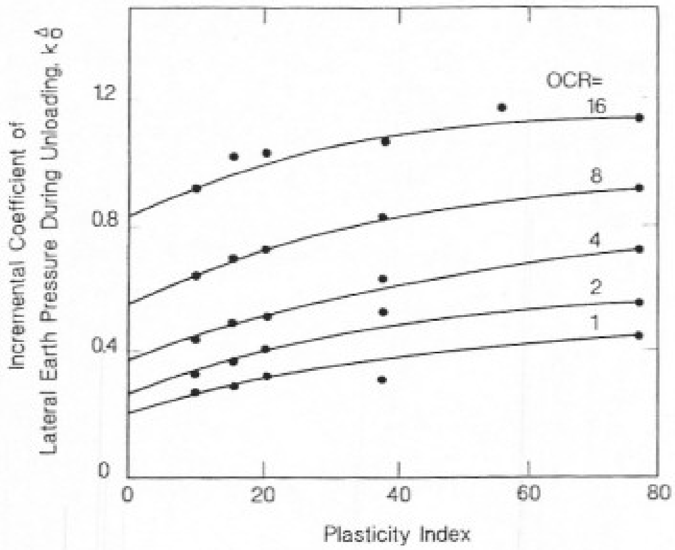

FIG. 9—*Correlations between K_0^Δ and plasticity index (PI) for various values of overconsolidation ratios (Clough and Duncan 1969).*

the same point. These values are not equal because the unloading curve does not extend on a straight line through the origin. The values of K_0^Δ are shown in Fig. 9 as a function of the plasticity index and overconsolidation ratio.

The tangent modulus varies throughout the length of the unload-reload portion of a consolidation curve with the highest value occurring at the initial portion of the curve where σ_3' is also the highest. The lowest value of tangent modulus occurs at the end of the rebound curve where σ_3' is the lowest. Therefore, Clough and Duncan (1969) recommended using a tangent modulus at the midpoint of the unload cycle and Eq 14 to estimate the average unload-reload modulus. The value of K_{ur} is then calculated using Eq 12, the primary loading exponent, and σ_3' at the midpoint of the unload cycle. Values of K_{ur} were calculated using this procedure and the consolidation test data in Fig. 5 and are shown in the top line of Table 4. It can be seen that the values ranged from 208 to 125 for the three consolidation tests considered. The average of these values, 167, underestimates the CID triaxial value of K_{ur}, 285, by a factor of 1.7.

Additional values of K_{ur} were calculated using various values of σ_3', tangent and secant moduli, and the consolidation test data in Fig. 5. It can be seen from Table 4 that a tangent modulus at the midpoint of the reload curve also resulted in a poor estimate of K_{ur}. In this case the value of σ_3' was also calculated at the midpoint of the reload curve. A secant modulus provided values of K_{ur} that were in much better agreement with the CID triaxial value. A secant modulus on the unload curve and an average value of σ_3' yielded the best estimate, 281, of the CID triaxial unload-reload modulus number. The average value of σ_3' equals the average of σ_3' at the beginning of the unload cycle and at the midpoint of the unload cycle.

At present, it is recommended that K_{ur} be determined using either a secant modulus on the unload curve and an average value of σ_3' or by multiplying the value of K_{ur} determined using Clough and Duncan's (1969) procedure by 1.7. It is also recommended that the consolidation specimen be unloaded such that multiplying the vertical effective stress at the midpoint of the unload cycle by the corresponding K_0 yields a value which is approximately equal to the desired value of σ_3'.

Conclusions and Recommendations

This paper presents a comparison of the effective stress hyperbolic stress-strain parameters determined from CID triaxial tests and consolidation and direct shear tests. The comparison revealed that the results of consolidation and direct shear tests underestimate the modulus number determined from CID triaxial test results. Since all the specimens were undergoing primary loading (loading beyond the preconsolidation pressure), the modulus exponent was approximately equal to unity for both the CID triaxial and consolidation tests.

The following procedure is recommended for estimating the initial tangent modulus and thus the modulus number from the results of consolidation and direct shear tests. At present, this procedure has been verified for only the medium to high plasticity clayey slopewash described herein. Therefore, this procedure should be used for normally consolidated soils with index properties similar to the slopewash. Additional soils are being tested to generalize this procedure:

1. Determine the average value of the failure ratio from the shear stress–horizontal displacement curves obtained from direct shear tests.
2. Calculate E_i for use in Eq 6 using a tangent modulus at the end of each load increment in the consolidation test and the void ratio at the beginning of the load increment. All values of E_i should be calculated in the primary loading region.
3. The resulting modulus number should be multiplied by 1.9

TABLE 4—*Effect of modulus and lateral earth pressure coefficient on the unload-reload modulus number from consolidation tests.*

Curve	Modulus	[b]σ_3'	Con #1 (Unload 1035 to 520 kPa), K_{ur}	Con #2 (Unload 765 to 383 kPa), K_{ur}	Con #3 (Unload 3640 to 1820 kPa), K_{ur}	Average, K_{ur}
[a]Unload	Tangent	Mid	208	170	125	167
Reload	Tangent	Mid	N/A	68	18.4	43
Unload	Secant	Init	234	281	175	230
Unload	Secant	Mid	378	454	284	372
Unload	Secant	Ave	289	347	207	281

[a]Clough and Duncan's (1969) procedure.
[b]Ave = σ_3' using average of σ_3' at initial and midpoint of unload cycle; Initial = σ_3' using K_0 and σ_v' at initial unloading point; Mid = σ_3' using K_0 and σ_v' at midpoint of unloading cycle; σ_v' = vertical effective stress.

to provide a reasonable estimate of the CID triaxial modulus number.

The consolidation and direct shear test results also provided poor estimates of the bulk modulus number and exponent determined from the CID triaxial test results. It is recommended that the consolidation and direct shear values of K_b and m be multiplied by 3.2 and 0.5, respectively, to provide a reasonable estimate of the CID triaxial values. The differences in the modulus and bulk modulus numbers are probably due to the different boundary conditions and stress paths inherent in the consolidation and CID triaxial tests. A procedure for estimating the unload-reload modulus number from an unload curve in a consolidation test is also presented.

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