Hyperbolic Stress-Strain Parameters for Silts

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Abstract: The hyperbolic stress-strain model has been shown to be valid for modeling the nonlinear stress-strain behavior of soils prior to failure in soil-structure interaction analyses. However, guidelines for selecting hyperbolic stress-strain parameters for silts have not been published to date. To fill this need, a series of isotropically consolidated-drained and consolidated-undrained triaxial tests were performed on freshly deposited, normally consolidated silts and clayey silts to provide guidance for selecting hyperbolic parameters for these materials. The effect of clay mineral content and dry unit weight on the hyperbolic parameters was investigated by reconstituting specimens at clay mineral contents of 0, 10, 20, 30, and 50% by dry weight and at standard Proctor relative compactions of 85, 90, 95, and 100%. At low clay mineral contents, the normally consolidated silt exhibited dilative shear behavior while a contractive behavior was observed at high clay mineral contents. The transition from dilative to contractive shear behavior is controlled by clay mineralogy. Due to the dilative behavior of the normally consolidated silt mixtures, failure criteria of maximum deviator stress and maximum pore-water pressure were used to obtain the Mohr-Coulomb shear strength parameters from the drained and undrained triaxial tests.

INTRODUCTION

The finite-element method provides the best approximation of the complex interaction that occurs between soils and structures. One analysis procedure models the nonlinear stress-strain relation of each soil element using an incremental construction procedure (Ebeling et al. 1990; Filz et al. 1991). The modulus of each soil element is adjusted during each loading increment in accordance with the total stresses accrued within each element. The constitutive relation used for each soil element is Hooke's law. The incremental changes in stress are related to the incremental strains through a linear relation. This relation is defined by Young's modulus and the bulk modulus for each soil element. The nonlinear stress-strain relation prior to failure is modeled using a hyperbolic curve as suggested by Konder (1963).

The hyperbolic relation between stress and strain developed by Duncan and Chang (1970), is defined in terms of an initial modulus E_i; the soil shear strength $(\sigma_1' - \sigma_3')_f$, and a reduction factor R_f. The reduction factor defines the deviatoric stress on the hyperbolic curve $(\sigma_1 - \sigma_3)_{ult}$. The initial modulus is defined by the following equation (Janbu 1963):

$$E_i = P_a K \left(\frac{\sigma_3'}{P_a}\right)^n \quad \text{(1)}$$

where K = modulus number; n = modulus exponent; σ_3' = effective confining pressure; and P_a = atmospheric pressure (101.3 kPa), which is used

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to make K and n nondimensional. The tangent modulus E_t for a given iteration is accordingly defined as

$$E_t = E_i(1 - S_L R_f)^2 \quad (2)$$

where the mobilized shear strength is equal to stress level S_L

$$S_L = \frac{(\sigma_1' - \sigma_3')}{(\sigma_1' - \sigma_3')_f} \quad (3)$$

and

$$(\sigma_1' - \sigma_3')_f = \frac{(2c' \cos \phi' + 2\sigma_3' \sin \phi')}{(1 - \sin \phi')} \quad (4)$$

where c' = intercept of the linear failure envelope; and ϕ' = slope of the linear failure envelope. If the Mohr-Coulomb failure envelope is nonlinear, the following expression (Duncan et al. 1978) can be used to model the variation of ϕ' with σ_3':

$$\phi' = \phi_0 - \Delta\phi \log\left(\frac{\sigma_3'}{P_a}\right) \quad (5)$$

where ϕ_0 = value of ϕ' when σ_3' equals P_a and $\Delta\phi$ is the reduction in ϕ' for an order-of-magnitude increase in σ_3', e.g., 100 kPa to 1,000 kPa.

The first term in (2), E_i, accounts for the affect of confining stress on the value of E_t assigned to each soil element for a given iteration, while the second term reflects the level of shear taking place within the element using the stress level. The stress level ranges in value from zero, corresponding to an isotropic stress state, to a value equal to unity, corresponding to the complete mobilization of the shear resistance. It should be noted that the hyperbolic relations can also be written in terms of total stresses.

The second elastic parameter used to define the material behavior is the bulk modulus B. The value of the bulk modulus is defined as the ratio of the change in mean principal stress to the change in volumetric strain ε_v

$$B = \frac{(\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3)}{3\varepsilon_v} \quad (6)$$

Since many soils exhibit nonlinear and stress-dependent volume change characteristics, a stress-dependent bulk modulus is required. Duncan et al. (1978) approximated this behavior using the following equation:

$$B = P_a K_b \left(\frac{\sigma_3'}{P_a}\right)^m \quad (7)$$

where K_b = bulk modulus number; and m = bulk modulus exponent.

In summary, eight parameters are employed in the hyperbolic stress-strain relationships developed by Duncan et al. (1978) for primary or virgin loading. These eight parameters are: K, n, K_b, m, c', ϕ_0, ϕ', $\Delta\phi$, and R_f. Duncan et al. (1978 and 1980) used the results of isotropically consolidated-drained and consolidated-undrained triaxial tests on compacted granular and clayey soils to provide guidance when selecting hyperbolic parameters for nonlinear finite-element analyses. However, guidance on selecting hyperbolic stress-strain parameters for silts and clayey-silts was not included.

OBJECTIVES

The main objectives of the present research were to investigate the stress-strain behavior of silts and present guidelines for estimating the hyperbolic stress-strain parameters for freshly deposited, normally consolidated silts and clayey-silts. This was accomplished by conducting isotropically consolidated-drained (ICD) and isotropically consolidated-undrained (ICU) triaxial tests on eight groups of reconstituted silt and clayey-silt mixtures. Table 1 presents the eight silt mixtures used in the present study and the triaxial tests performed on each mixture. The effect of clay mineral content was studied by reconstituting silt specimens containing 0, 10, 20, 30, and 50% clay mineral by dry weight. The effect of dry unit weight on the stress-strain behavior of each clay mineral content was investigated by using relative compactions of 85, 90, 95, and 100% of the maximum standard Proctor dry unit weight. The effect of clay mineralogy was also investigated by using kaolinite and montmorillonite to fabricate the clay-silt specimens. The present paper describes the laboratory testing program, stress-strain behavior of silt mixtures, and guidelines for selecting hyperbolic parameters for silts.

LABORATORY TESTING PROGRAM

Silt Tested

The silt used in the present study was obtained from a 12.2 m high bluff composed of Mississippi loess and located at the U.S. Army Engineer Waterways Experiment Station in Vicksburg, Miss. This wind-blown loess has a uniform gradation with a median grain size of 0.02 mm. The undisturbed loess is highly structured, which allows the 12.2 m high bluff to maintain a nearly vertical face. The loess was excavated from the bluff using a pick and shovel, and thus the natural soil structure or fabric was destroyed during sampling. Particle-size analyses revealed that the light-brown loess has a natural clay fraction (percentage finer than 0.002 mm) of 9–11%. Approximately 2–3% of the loess is fine sand, shells, and organic particles that did not pass U.S. Standard Sieve No. 200. Therefore, 85–90% of the loess consists of silt size particles (0.075 to 0.002 mm). Scanning-electron-microscope photographs revealed that the silt particles are subangular to subrounded.

TABLE 1. Triaxial Tests Performed and Index Properties of Silt Mixtures

Silt-clay mixture (1)	Triaxial tests performed[a] (2)	Specific gravity (3)	Liquid limit (4)	Plastic limit (5)	Plasticity index (6)	USCS symbol[b] (7)	Clay fraction (percent) (8)
0% Kaolinite	ICD and ICU	2.7	27	NP	NP	ML	0
10% Kaolinite	ICD and ICU	2.69	29	23	6	CL-ML	9
20% Kaolinite	ICD	2.68	30	22	8	CL	14
30% Kaolinite	ICD and ICU	2.67	30	20	10	CL	19
50% Kaolinite	ICD and ICU	2.62	38	22	16	CL	33
10% Montmorillonite	ICD	2.71	55	29	26	CH	11
30% Montmorillonite	ICD	2.71	152	26	126	CH	21
50% Montmorillonite	ICD	2.72	186	53	133	CH	39

[a]Standard Proctor relative compactions of 85, 90, 95, and 100% were used for each kaolinite mixture. A standard Proctor relative compaction of only 100% was used for the montmorillonite mixtures.

[b]USCS = Unified soil classification system.

The naturally occurring clay fraction of 9–11% was removed using a sedimentation process (Stark et al. 1991). After air drying, the silt was passed through the No. 200 sieve to remove the fine sand, shells, and organics present in the natural loess. After sieving approximately 200 kg, or 56%, of the original 360 kg of the Mississippi loess remained. All of the processed silt was combined and thoroughly mixed to ensure a uniform sample. The clay-size fraction of 10 specimens obtained from the processed silt ranged from 0.8 to 1.2%. This small clay size fraction was neglected and the silt was assumed to have zero clay size particles.

To characterize the effect of clay mineralogy on the stress-strain and strength behavior of silt, two different clay minerals were combined with the processed silt. Commercially available kaolinite and montmorillonite were mixed with the silt to obtain samples representative of low and high plasticity silt. The manufactured kaolinite and Wyoming bentonite, i.e., montmorillonite, were processed through sieve No. 200 and then mixed with the silt to obtain the clay-silt mixtures.

Sample Preparation

Based on research by Lupini et al. (1981), Mulilis et al. (1977), and Seed et al. (1964), the silt mixtures were fabricated using a dry mixing technique. The samples were composed of the processed silt, and the following clay mineral percentages: 0, 10, 20, 30, and 50. These percentages were based on the dry weight of silt and clay. The dry mixing process involved mixing 4.5–6.8 kg of silt and clay by hand in a 15.2-L bucket, and then adding the appropriate amount of distilled and deionized water. Any clumps of soil that developed during mixing were processed through the No. 40 sieve.

To study the effect of unit weight on the stress-strain behavior and the Mohr-Coulomb strength parameters, the silt mixtures were tested at dry unit weights corresponding to standard Proctor relative compactions of 85, 90, 95, and 100%. To simplify the effect of compaction water content, and thus soil fabric, the specimens were compacted at the optimum water content for each relative compaction. The optimum water-content criterion is also consistent with the test results used by Duncan et al. (1978, 1980) to develop their guidelines for hyperbolic stress-strain parameters. To determine the optimum water content for each relative compaction, the line of optimums for each silt mixture was obtained from the results of modified Proctor and standard Proctor compaction tests. Fig. 1 illustrates the technique used to obtain the optimum water content for each relative compaction.

Determination of Triaxial Consolidation Pressures

The main objective of the present study was to characterize the stress-strain behavior of freshly deposited, normally consolidated silts and clayey silts. Therefore, laboratory triaxial tests had to be performed at consolidation pressures that ensured normally consolidated behavior. To estimate the consolidation pressures required for normally consolidated behavior, an oedometer test was conducted to estimate the preconsolidation pressure for each standard Proctor relative compaction value and each clay mineral percentage to be tested.

The oedometer specimens were compacted directly into a fixed oedometer ring at the appropriate dry unit weight and water content. A modified Harvard compaction apparatus was used to compact the specimens. This apparatus controls the height of each lift and thus the amount of soil compacted into each lift. Therefore, the appropriate amount of soil was weighed and compacted in four lifts to create the 1-cm-thick test specimens with the desired dry unit weight or relative compaction. The top of each lift was scarified before the next lift was placed to ensure an adequate bond between lifts.

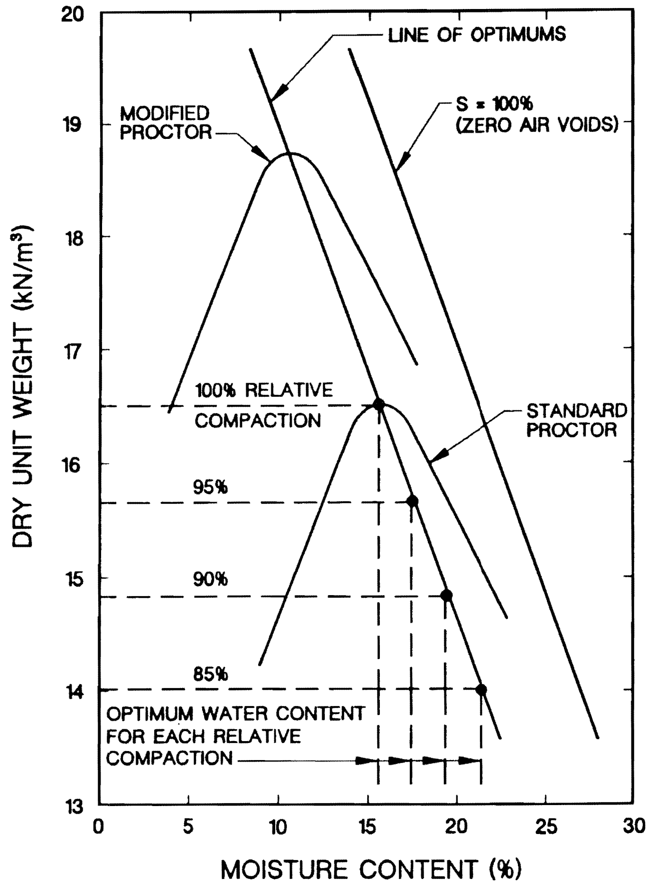

FIG. 1. Line of Optimums and Compaction Criteria

Based on the oedometer test results, the estimated preconsolidation pressures of the silt mixtures ranged from 30 kPa for the 50% clay mixtures to 1,530 kPa for 0% clay mixtures (Table 2). Stark et al. (1991) present the oedometer test results on the eight silt mixtures. The oedometer test results were also used to verify the hyperbolic parameters using the nonlinear soil-structure interaction program FEADAM84 (Duncan et al. 1984) to simulate one-dimensional compression.

Compaction of Triaxial Specimens

To obtain the dry unit weight corresponding to standard Proctor relative compactions of 85, 90, 95 and 100%, the silt mixtures were compacted in five 2.5 cm lifts in a 15.2 cm diameter Proctor compaction mold. To compact the appropriate weight of soil into each lift, the height of each lift was fixed by adjusting the depth to which the tamper could penetrate. The depth was

TABLE 2. Triaxial Consolidation Pressures and Compressibility Parameters of Silt-Clay Mixtures

Percent clay mineral[a] (1)	Standard Proctor relative compaction (2)	Maximum preconsolidation pressure (kPa) (3)	Range of triaxial consolidation pressure (kPa) (4)	Compression index[c] (5)	Recompression index[c] (6)	Modified compression index[d] (7)	Modified recompression index[d] (8)
0% Kao	100	1,530	1,120–1,740	0.071	0.0041	0.042	0.0024
0% Kao	95	1,000	755–1,725	0.081	0.0045	0.046	0.0025
0% Kao	90	820	765–1,585	0.105	0.0067	0.055	0.0035
0% Kao	85	N/A[b]	N/A[b]	N/A[b]	N/A[b]	N/A[b]	N/A[b]
10% Kao	100	1,410	940–1,630	0.070	0.0069	0.043	0.0043
10% Kao	95	880	1,120–1,665	0.081	0.0093	0.048	0.0055
10% Kao	90	170	295–1,475	0.114	0.0099	0.064	0.0056
10% Kao	85	50	300–1,390	0.120	0.0101	0.067	0.0056
30% Kao	100	920	980–1,685	0.122	0.0117	0.078	0.0075
30% Kao	95	130	795–1,485	0.124	0.0118	0.082	0.0078
30% Kao	90	40	395–1,520	0.139	0.0120	0.091	0.0080
30% Kao	85	30	300–1,660	0.148	0.0131	0.092	0.0081
50% Kao	100	580	880–1,670	0.106	0.0092	0.007	0.0006
50% Kao	95	100	785–1,475	0.130	0.0114	0.077	0.0067
50% Kao	90	80	390–1,275	0.207	0.0174	0.117	0.0098
50% Kao	85	30	350–1,280	0.214	0.0196	0.122	0.0112
10% Mont	100	1,340	1,080–1,620	0.079	0.0082	0.054	0.0055
30% Mont	100	620	990–1,620	0.203	0.0206	0.116	0.0118
50% Mont	100	610	840–1,485	0.322	0.0280	0.162	0.0141

[a]Kao = Kaolinite; Mont = Montmorillonite.
[b]N/A = Not available.
[c]Compression and recompression indices from void ratio-effective stress relationships.
[d]Modified compression and recompression indices from axial strain-effective stress relationships.

set using a series of 2.5 cm spacer blocks. Each lift was tamped until all of the soil was compacted into the predetermined lift height. The height of the tamper was then adjusted using another spacer block and the next lift was compacted.

To facilitate saturation of the triaxial specimens, carbon dioxide gas was introduced into the compaction mold as the soil was being rained in. The carbon dioxide displaced air during the raining process and dissolved during back-pressure saturation, which reduced the time required to obtain full saturation. The top of each lift was scarified before the next lift was placed to ensure an adequate bond between lifts. Each compaction mold yielded four triaxial specimens with a similar compaction history and thus comparable shear strength and stress-strain characteristics. The compacted sample was slowly extruded from the compaction mold using a hydraulic jack. The sample was cut into four sections using a fine wire saw. The four sections were trimmed into triaxial specimens having a diameter of 3.6 cm and a length of 9.0 cm.

The strain rate used for the ICD and ICU triaxial tests was determined using the procedure described by Gibson and Henkel (1954), the coefficient of consolidation evaluated during consolidation of the triaxial specimens, and the results of the oedometer tests. The axial strain rate used for the ICD triaxial tests on the 0% and 10% kaolinite-silt mixtures was 0.05%/min or 0.0046 cm/min. Strain rates of 0.013%/min and 0.01%/min were used for the 30 and 50% kaolinite-silt mixtures, respectively. The ICD triaxial tests on the montmorillonite-silt mixtures were performed using a strain rate of 0.002%/min. The ICU triaxial tests on the kaolinite-silt mixtures were conducted at a strain rate of 0.086%/min.

Saturation of the triaxial specimen was confirmed using the pore-pressure coefficient B (Skempton 1954). In accordance with Black and Lee (1973) and Bishop and Henkel (1962), a B-value greater than 95% and greater than 99.7% were required for the ICD and ICU triaxial tests, respectively. If the B-valve did not meet this criterion, the cell and back pressure were incrementally increased until they were achieved. The rate and size of the increment were controlled to ensure that no part of the specimen was overconsolidated. It was found that the 0% and 10% kaolinite specimens required a back pressure of approximately 96 kPa and about one day of back pressure to achieve the saturation criteria. The 30% and 50% kaolinite specimens required back pressures as high as 480–570 kPa and durations of up to 7 days to achieve the saturation criteria.

HYPERBOLIC STRESS-STRAIN PARAMETERS FOR KAOLINITE-SILT MIXTURES

Drained Stress-Strain Behavior of Silts

Table 3 presents the hyperbolic stress-strain parameters obtained from the ICD triaxial tests on the kaolinite-silt mixtures. The failure criterion used to estimate the drained Mohr-Coulomb shear strength parameters was the maximum deviator stress. The initial estimate of the hyperbolic parameters was obtained using the procedure recommended by Duncan et al. (1980) in which the deviator stresses at 70% and 95% of the maximum deviator stress are used to estimate the initial tangent modulus. The initial values of modulus number and modulus exponent were varied to obtain the best geometric agreement between the measured and hyperbolic stress-strain relations. Since the initial tangent modulus is an important parameter in soil-structure interaction analyses, the best geometric agreement was sought

TABLE 3. Drained Hyperbolic Stress-Strain Parameters for Kaolinite-Silt Mixtures from ICD Triaxial Tests

Percent clay mineral[a] (1)	Standard Proctor relative compaction (2)	Average initial dry unit weight (kN/m³) (3)	Average initial water content (percent) (4)	Range of triaxial consolidation pressure (kPa) (5)	Effective stress cohesion (kPa) (6)	Effective stress friction angle (degrees) (7)	Modulus number K (8)	Modulus exponent n (9)	Bulk modulus number K_b (10)	Bulk modulus exponent m (11)	Failure ratio (R_f) (12)
0% Kao	100	98	27	1,120–1,740	0	40	270	1.0	115	1.00	0.75
0% Kao	95	97	27	755–1,725	0	37	150	1.0	65	1.00	0.70
0% Kao	90	96	28	765–1,585	0	35	120	1.0	50	1.00	0.65
0% Kao	85	N/A[b]	N/A[b]	N/A[b]	N/A[b]	N/A[b]	N/A[b]	N/A[b]	N/A[b]	N/A[b]	N/A[b]
10% Kao	100	104	23	940–1,630	0	37	240	1.0	85	1.00	0.85
10% Kao	95	103	23	1,120–1,665	0	35	125	1.0	55	1.00	0.80
10% Kao	90	103	23	295–1,475	0	34	100	1.0	40	1.00	0.75
10% Kao	85	102	24	300–1,390	0	33	75	1.0	30	1.00	0.70
30% Kao	100	115	17	980–1,685	0	33	105	1.0	35	1.00	0.80
30% Kao	95	115	17	795–1,485	0	31	70	1.0	30	1.00	0.75
30% Kao	90	114	17	395–1,520	0	30	65	1.0	25	1.00	0.70
30% Kao	85	114	18	300–1,660	0	29	60	1.0	20	1.00	0.65
50% Kao	100	111	18	880–1,670	0	28	65	1.0	30	1.00	0.75
50% Kao	95	109	19	785–1,475	0	27	60	1.0	25	1.00	0.70
50% Kao	90	108	20	390–1,275	0	26	55	1.0	20	1.00	0.65
50% Kao	85	107	21	350–1,280	0	25	50	1.0	15	1.00	0.60

[a]Kao = Kaolinite.
[b]N/A = Not available.

at the initial portion of the measured stress-strain relation. It should be noted that the specimen containing 0% kaolinite could not be tested because it appeared to liquefy at a relative compaction of 85%.

Fig. 2 presents a typical comparison of the actual and hyperbolic stress-strain relations for the 30% and 50% kaolinite-silt mixtures. These data were obtained from the ICD triaxial test conducted on a 50% kaolinite-silt specimen at a relative compaction of 85% and an isotropic consolidation pressure σ'_{3c}, of 550 kPa. It can be seen that the silt mixtures with a high kaolinite content exhibit a contractive shear behavior. As a result, the hyperbolic stress-strain model provides an excellent representation of the deviator stress and volumetric strain relations for the 30% and 50% kaolinite-silt mixtures at axial strains less than 20%.

Fig. 3 presents a typical comparison of the actual and hyperbolic stress-strain curves for the 0% and 10% kaolinite-silt mixtures. These data were obtained from the ICD triaxial test conducted on a 10% kaolinite-silt specimen at a relative compaction of 100% and an isotropic consolidation pressure of 1,275 kPa. It can be seen that the 0% and 10% kaolinite-silt mixtures

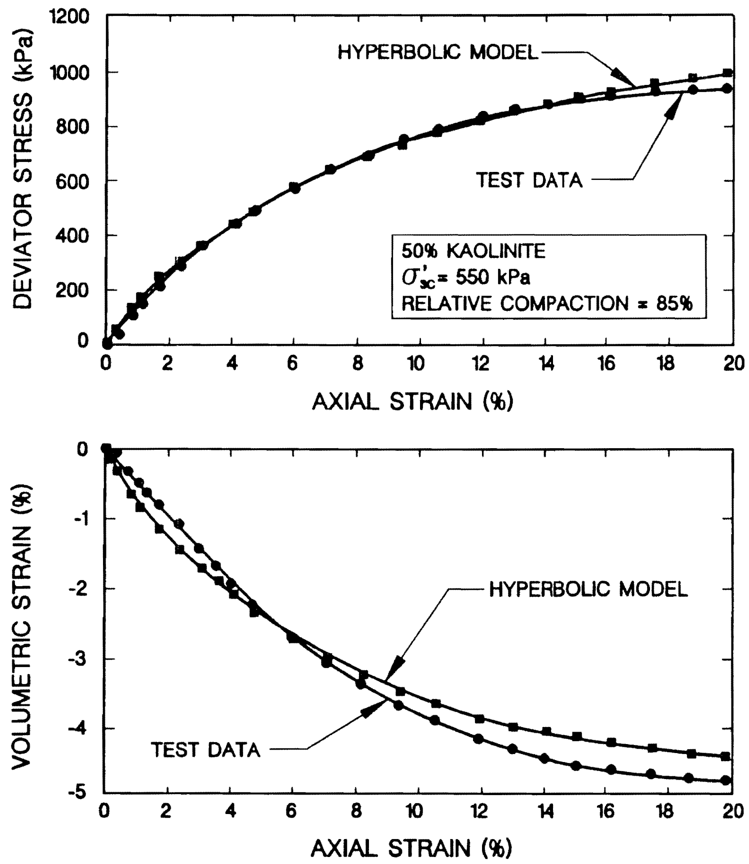

FIG. 2. Typical Comparison of Measured and Hyperbolic Drained Stress-Strain Curves for 30 and 50% Kaolinite-Silt Mixtures

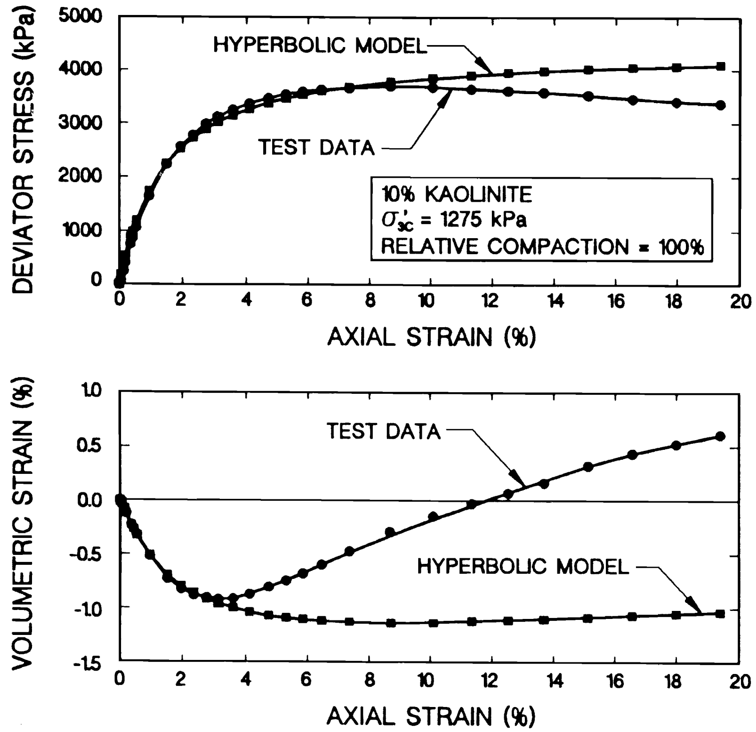

FIG. 3. Typical Comparison of Measured and Hyperbolic Drained Stress-Strain Curves for 0 and 10% Kaolinite-Silt Mixtures

exhibited a dilative behavior. The magnitude of dilation in the 0% and 10% kaolinte mixtures decreased as the consolidation pressure increased. Dilation has also been observed in triaxial tests on normally consolidated silt by Penman (1953), Schultze and Horn (1965), Schultze and Odenhall (1967), Nacci and D'Andrea (1976), Torrey (1982), Wang et al. (1982), Kuerbis et al. (1985), Ladd et al. (1985), and Fleming and Duncan (1990). The silt specimens tested during the present study were normally consolidated, and thus the observed dilation appears to be characteristic of normally consolidated silts.

Since the hyperbolic stress-strain model does not account for dilation, the hyperbolic parameters were varied to provide the best approximation of the deviator stress-strain relation at small axial strains. It can be seen that the hyperbolic model provides an excellent approximation of the deviator stress-strain relation for axial strains up to 8%. At axial strains greater than 8%, the hyperbolic model overpredicts the deviator stress. This is due to increasing the modulus number to account for the increase in initial tangent modulus caused by the dilative response. Due to dilation, the hyperbolic model also provides a poor representation of the volumetric strain behavior at axial strains greater than approximately 3%.

Mitchell (1976) reported that the peak effective stress friction angle of pure kaolinite ranges from 26 to 30 degrees. Therefore, it may be concluded from Table 3 that kaolinite controls the effective stress friction angle of the

30% and 50% kaolinite-silt mixtures. The effective stress friction angle of the 0% and 10% kaolinite-silt mixture ranges from 33 to 40 degrees, and thus it appears that the shear strength of the 0% and 10% kaolinite mixtures is controlled by the silt. The transition from "silt behavior," i.e., dilation, to "clay behavior," i.e., contraction, appears to occur at a kaolinite content between 10% and 30%. As a result, three additional ICD triaxial tests at a kaolinite content of 20% were conducted to estimate the kaolinite content at the transition point. The triaxial tests at a kaolinite content of 20% exhibited a slight contractive shear behavior. Therefore, it was concluded that normally consolidated silts exhibit a transition from dilative to contractive shear behavior at a kaolinite-silt content between 15 and 20%.

The shear behavior transition point also appears to be a function of clay mineralogy. The ICD triaxial tests on montmorillonite-silt mixtures described subsequently, showed that the transition point occurs at a montmorillonite content less than 10%. Thus, clay mineralogy or plasticity plays an important role in the shear behavior of silts. An increase in plasticity appears to reduce the percentage of clay mineral required to develop a contractive shear behavior.

In summary, the hyperbolic stress-strain model will provide an excellent representation of the drained stress-strain behavior of silts if the clay mineral content is greater than or equal to 20%. If the predominant clay mineral is montmorillonite, the hyperbolic model will be suitable if the clay mineral content is greater than or equal to 10%. At lower clay mineral contents, normally consolidated silts exhibit dilation and the hyperbolic parameters presented herein only represent the actual deviator stress and volumetric strain relationships for axial strains less than 8% and 3%, respectively.

Drained Hyperbolic Stress-Strain Parameters of Silts

Fig. 4 illustrates the influence of relative compaction and clay mineral content on the drained modulus number of the kaolinite-silt mixtures. It can be seen that the modulus number ranged from 120 to 270 for the 0% kaolinite-silt mixture and only 50 to 65 for the 50% kaolinite mixture. It can also be seen that there is a substantial decrease in modulus number at a kaolinite content between 10% and 30%. This indicates a large decrease in stiffness, which also suggests a transition from dilative to contractive shear behavior. It can be seen that the modulus number increases significantly only at relative compactions greater than 95%. The effect of relative compaction also decreases as the kaolinite content increases. These data suggest that field compaction should exceed 95% of the maximum standard Proctor dry unit weight to obtain a substantial increase in modulus number.

The modulus exponent is equal to unity for all of the kaolinite-silt mixtures shown in Fig. 4. This is in good agreement with the fact that the triaxial specimens are freshly deposited and normally consolidated. In fact, it has been shown by Clough and Duncan (1969) that the modulus exponent is equal to unity if the specimen is unstructured and normally consolidated. The failure ratio for the kaolinite-silt mixtures varied from 0.6 to 0.85 and appeared to increase slightly with increasing relative compaction and decrease slightly with increasing kaolinite content. A parametric study revealed that small variations in the failure ratio did not significantly affect the shape of the stress-strain curve predicted by the hyperbolic stress-strain model.

Fig. 5 shows that relative compaction and kaolinite content have a similar affect on drained bulk modulus number as that observed for the drained modulus number. The initial estimate of the bulk modulus number and

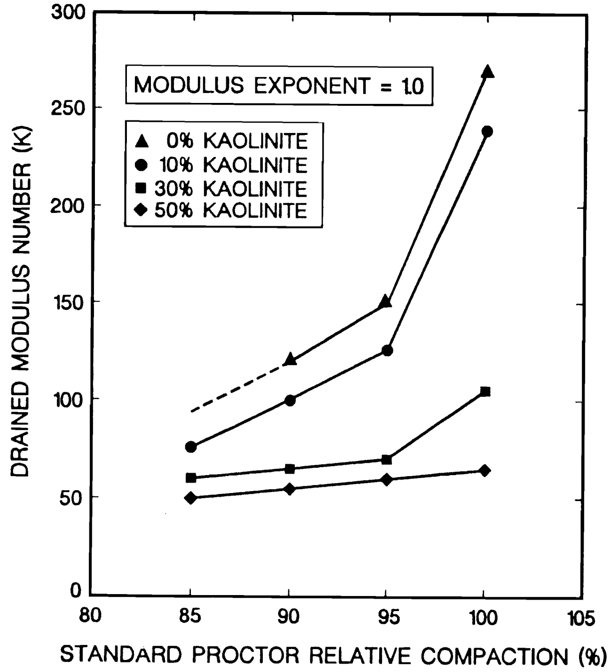

FIG. 4. Effect of Clay Mineral Content and Relative Compaction on Drained Modulus Number of Kaolinite-Silt Mixtures

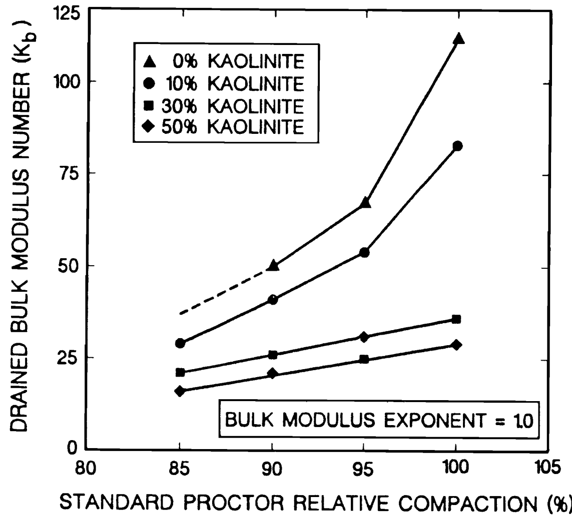

FIG. 5. Effect of Clay Mineral Content and Relative Compaction on Drained Bulk Modulus Number of Kaolinite-Silt Mixtures

exponent were obtained using the volumetric strain at 70% of the maximum deviator stress as recommended by Duncan et al. (1980). The values of bulk modulus number and bulk modulus exponent were then varied to obtain the best geometric agreement between the measured and hyperbolic volumetric strain relations. The bulk modulus number shows a significant increase between a kaolinite content of 10% and 30%. This also suggests a transition from dilative to contractive shear behavior at or near a kaolinite content of 20%. The bulk modulus exponent is equal to unity for all of the kaolinite-silt mixtures shown in Fig. 5.

In summary, Table 3 provides a database of effective stress shear strength and hyperbolic stress-strain parameters for normally consolidated silts and clayey silts. The hyperbolic and shear strength parameters for natural silt deposits can be estimated from Table 3 using the in situ dry unit weight and water content. It should be noted that the hyperbolic stress-strain model does not account for dilation. Therefore, the dilative behavior observed in the deviator stress and volumetric strain relations for low (0%–20%) kaolinite contents will not be accurately modeled. Therefore, the hyperbolic parameters in Table 3 for the low kaolinite contents should be used for soil-structure interaction analyses in which the strains are less than approximately 8%. If the strains are greater than 8%, bulk modulus values corresponding to a Poisson's ratio of nearly 0.5 can be used for the low kaolinite contents. The hyperbolic parameters in Table 3 for the high (30%–50%) kaolinite contents provide an excellent representation of the measured stress-strain behavior and can be used for soil-structure interaction analyses involving strains to 20%.

Undrained Hyperbolic Stress-Strain Parameters of Silts

Table 4 presents the hyperbolic stress-strain parameters obtained from the ICU triaxial tests on the kaolinite-silt mixtures. These parameters were obtained using the total stress friction angle and the best geometric agreement between the measured and hyperbolic stress-strain curves. Interpretation of the undrained triaxial tests was difficult because the dilative behavior of the silt mixtures caused the deviator stress to increase with increasing strain. Therefore, the larger the axial strain corresponding to the failure criterion, the larger the deviator stress and Mohr-Coulomb strength parameters. A number of failure criteria were considered including maximum deviator stress, maximum principal stress ratio, a limiting axial strain, maximum pore-water pressure, a pore-pressure coefficient A (Skempton 1954) equal to zero, and the stress path reaching the K_f line. It was found that the maximum pore-water pressure was satisfied at an axial strain of 1–3%, which is larger than the strains computed within the soil elements in most soil-structure interaction analyses involving nonyielding backfills. The other failure criteria were usually satisfied at larger axial strains, and thus yielded higher shear strength parameters due to the increase in deviator stress with increasing axial strains. As a result, the maximum pore-water pressure criterion was used to estimate the Mohr-Coulomb shear strength parameters from the ICU triaxial tests.

Fig. 6 presents a typical comparison of the actual and hyperbolic stress-strain relationships for the 30% and 50% kaolinite mixtures. This undrained triaxial data was obtained from an ICU triaxial test conducted on a 50% kaolinite mixture at a relative compaction of 95% and a consolidation pressure of 1,475 kPa. Due to the ductile stress-strain behavior of the high kaolinite mixtures, the hyperbolic model provides an accurate representa-

TABLE 4. Undrained Hyperbolic Stress-Strain Parameters for Kaolinite-Silt Mixtures from ICU Triaxial Tests

Percent clay mineral (1)	Standard Proctor relative compaction (2)	Average initial dry unit weight (kN/m^3) (3)	Average initial water content (percent) (4)	Range of triaxial consolidation pressure (kPa) (5)	Total stress cohesion (kPa) (6)	Total stress friction angle (degrees) (7)	Modulus number K (8)	Modulus exponent n (9)	Failure ratio R_f (10)
0% Kao	100	15.2	27	1,080–1,570	0	19	450	1.0	0.65
0% Kao	95	14.9	28	585–1,590	0	18	400	1.0	0.60
0% Kao	90	14.7	29	835–1,585	0	16	350	1.0	0.55
0% Kao	85	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
10% Kao	100	16.7	22	1,320–1,620	0	18	425	1.0	0.45
10% Kao	95	16.2	23	985–1,565	0	17	375	1.0	0.55
10% Kao	90	16.0	24	500–1,620	0	16	350	1.0	0.60
10% Kao	85	15.7	25	400–1,660	0	15	300	1.0	0.70
30% Kao	100	17.8	18	885–1,470	0	16	400	1.0	0.90
30% Kao	95	17.3	20	300–1,230	0	15	350	1.0	0.90
30% Kao	90	17.0	21	395–1,520	0	13	325	1.0	0.95
30% Kao	85	16.8	22	300–1,280	0	12	270	1.0	0.95
50% Kao	100	16.3	22	640–1,375	0	15	250	1.0	0.80
50% Kao	95	16.2	23	690–1,475	0	14	240	1.0	0.85
50% Kao	90	16.0	24	725–1,275	0	13	230	1.0	0.90
50% Kao	85	15.9	25	350–1,280	0	12	210	1.0	0.90

[a]Kao = Kaolinite.
[b]N/A = Not available.

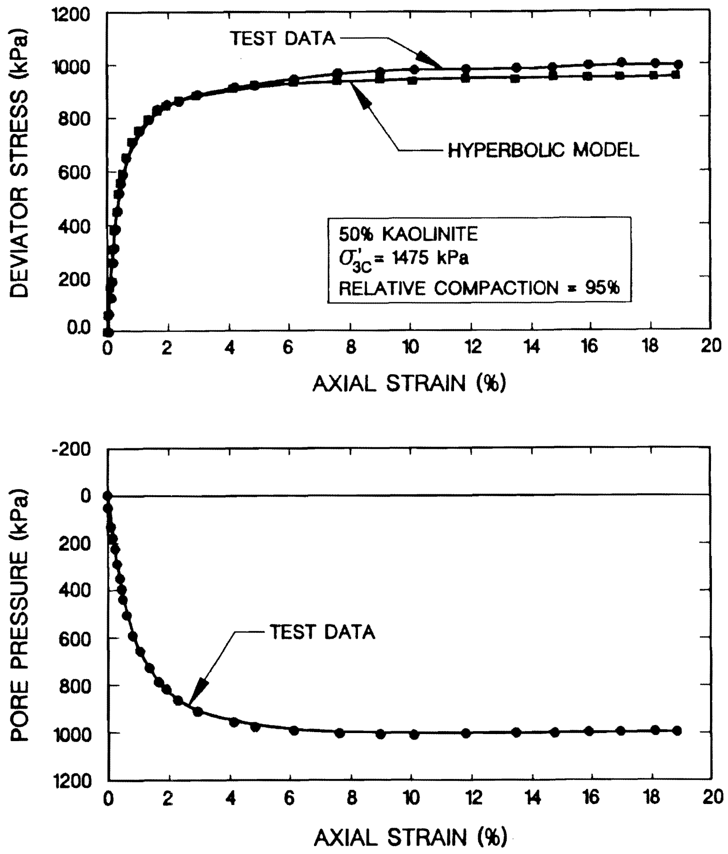

FIG. 6. Typical Comparison of Measured and Hyperbolic Undrained Stress-Strain Curves for 30 and 50% Kaolinite-Silt Mixtures

tion of the measured deviator stress-strain relation. It can also be seen that the shear-induced pore pressures are positive and become essentially constant at approximately 1,000 kPa.

It can be seen from Fig. 7 that the hyperbolic stress-strain model is in poor agreement with the measured deviator stress-strain relation for the 0% and 10% kaolinite-silt mixture at strains greater than 2%. This undrained triaxial data was obtained from an ICU triaxial test conducted on a 0% kaolinite mixture at a relative compaction of 95% and a consolidation pressure of 1,290 kPa. It can be seen that the normally consolidated silt mixtures exhibited an increase in deviator stress with increasing axial strain under undrained conditions. The pore-water pressure reaches a positive value of approximately 700 kPa and then decreases as the specimen dilates. This is due to the development of negative pore-water pressures through an axial strain of approximately 18%. Therefore, the deviator stress corresponding to a pore-water pressure of approximately 700 kPa was used to estimate the total stress friction angle in Table 4.

The undrained hyperbolic stress-strain parameters were determined by modeling the initial portion of the deviator stress-strain relation for the 0%

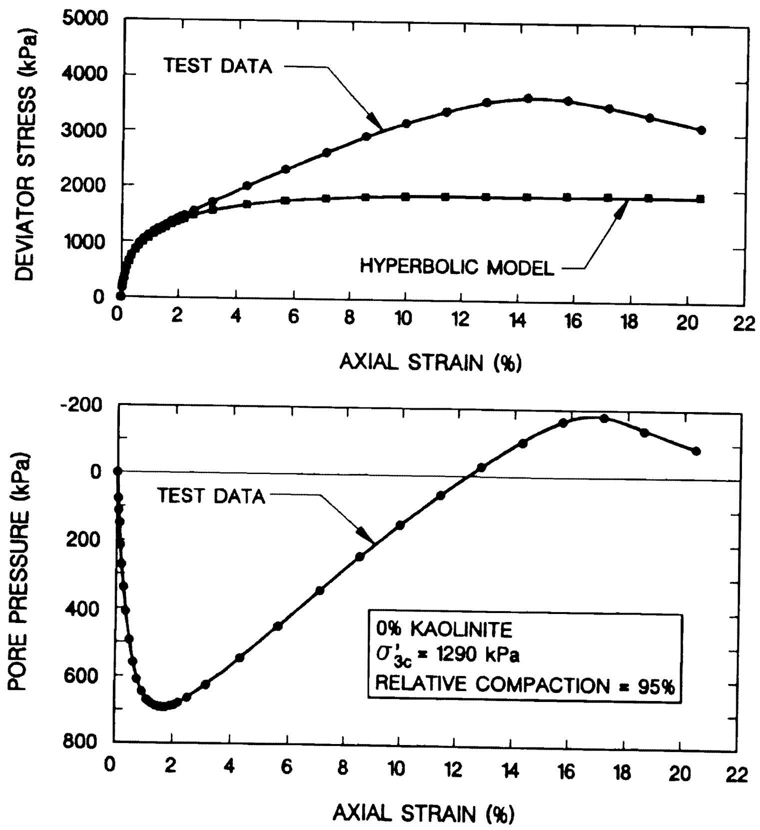

FIG. 7. Typical Comparison of Measured and Hyperbolic Undrained Stress-Strain Curves for 0 and 10% Kaolinite-Silt Mixtures

and 10% kaolinite mixtures. It can be seen that the hyperbolic model is in excellent agreement with the measured deviator stress-strain relation at axial strains less than 2% (Fig. 7). Since the hyperbolic model does not account for dilation, the geometric agreement at axial strains greater than 2% is poor. Therefore, the hyperbolic stress-strain parameters presented in Table 4 for the low kaolinite content mixtures (0% and 10%) should only be used for analyses involving strains less than approximately 2% to 5%.

Comparisons between the initial modulus calculated from Table 4 and Duncan et al. (1980) for the various silt mixtures revealed that the parameters in Table 4 yield steeper stress-strain curves. Possible explanations for the difference in stress-strain behavior include soil-type differences, the significantly higher consolidation pressures used in the present study, and the type of triaxial test. Duncan et al. (1980) used the results of isotropic unconsolidated-undrained triaxial tests, while the parameters in Table 4 were developed from the results of ICU triaxial tests.

In summary, Table 4 provides a database of undrained shear strength and hyperbolic stress-strain parameters for the various kaolinite-silt mixtures considered. The undrained hyperbolic stress-strain and shear-strength parameters for normally consolidated silt deposits can be estimated from Table 4 using the in situ dry unit weight, water content, and the applicable range of consolidation pressure. It should be noted that the hyperbolic stress-strain model does not account for dilation, and thus the hyperbolic parameters for the low kaolinite contents are only applicable for axial strains less than 2–5%. However, the hyperbolic parameters provide excellent agreement with the measured stress-strain behavior at kaolinite contents greater than 20%.

DRAINED HYPERBOLIC STRESS-STRAIN PARAMETERS FOR MONTMORILLONITE-SILT MIXTURES

A total of 10 ICD triaxial tests were performed on montmorillonite-silt mixtures. These tests were performed to determine the effect of clay mineralogy on the drained hyperbolic stress-strain parameters of silts and clayey silts. Only a standard Proctor relative compaction of 100% and the three clay mineral percentages, 10, 30, and 50% were used in the testing of the montmorillonite-silt mixtures. As expected, the montmorillonite-silt mixtures exhibited lower modulus numbers and shear strength parameters than the kaolinite-silt mixtures at all clay mineral contents. This is illustrated in Fig. 8 in which the drained modulus number for the montmorillonite-silt mixtures is significantly lower than the modulus number of the kaolinite-silt mixtures. The 10% montmorillonite-silt mixtures exhibited a contractive volume change behavior while the 10% kaolinite-silt mixtures exhibited a dilative behavior. This indicates that the transition point between dilative and contractive shear behavior occurs at a montmorillonite content less than 10% instead of 15–20% for the kaolinite-silt mixtures.

Table 5 summarizes the drained hyperbolic stress-strain parameters for the montmorillonite-silt mixtures. By comparing Tables 3 and 5, it can be

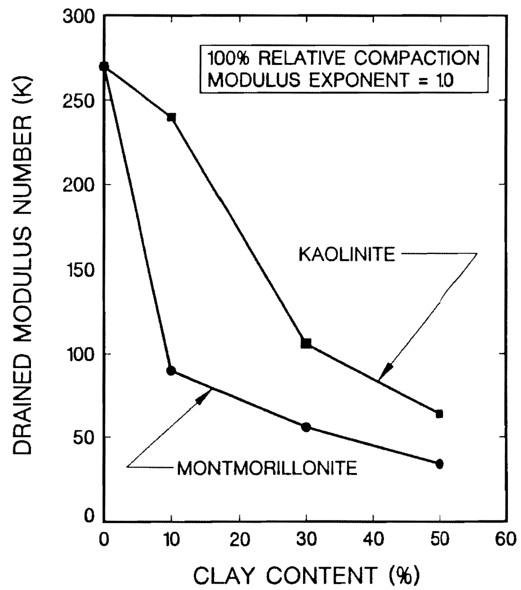

FIG. 8. Effect of Clay Mineralogy on Modulus Number of Silt Mixtures

TABLE 5. Drained Hyperbolic Stress-Strain Parameters for Montmorillonite-Silt Mixtures from ICD Triaxial Tests

Percent clay mineral[a] (1)	Standard Proctor relative compaction (2)	Average initial dry unit weight (kN/m^3) (3)	Average initial water content (percent) (4)	Range of triaxial consolidation pressure (kPa) (5)	Effective stress cohesion (kPa) (6)	Effective stress friction angle (degrees) (7)	Modulus number K (8)	Modulus exponent n (9)	Bulk modulus number K_b (10)	Bulk modulus exponent m (11)	Failure ratio R_f (12)
0% Mont	100	98	27	200–1,640	0	40	270	1.0	115	1.0	0.75
10% Mont	100	102	24	1,080–1,620	0	35	90	1.0	50	1.0	0.75
30% Mont	100	100	26	990–1,620	0	20	55	1.0	20	1.0	0.75
50% Mont	100	94	30	840–1,485	0	14	35	1.0	15	1.0	0.75

[a]Mont = Montmorillonite.

seen that the bulk modulus number for the montmorillonite-silt mixtures are also substantially lower than the corresponding values of the kaolinite-silt mixtures. The in situ dry unit weight, water content, and Table 5 can be used to estimate the hyperbolic and shear strength parameters for normally consolidated silt deposits that contain montmorillonite. The montmorillonite-silt mixtures exhibited contractive shear behavior for a clay mineral content of 10%. Therefore, the hyperbolic stress-strain and volume change model will probably provide an accurate representation of the measured stress-strain behavior when the montmorillonite content is greater than or equal to 5%.

EFFECT OF COMPACTION ON HYPERBOLIC STRESS-STRAIN PARAMETERS

Boscardin et al. (1990) present drained hyperbolic stress-strain parameters for a compacted sandy silt. The sandy silt has a liquid and plastic limit of 20 and 16, respectively, and classifies as a ML/CL-ML according to the unified soil classification system. The sandy silt was tested at standard Proctor relative compactions of 85, 90, and 95%. The ICD triaxial tests were performed at consolidation pressures less than 310 kPa. The specimens were compacted at the optimum water content and were not back-pressure saturated prior to shear. In summary, the specimens were partially saturated and overconsolidated due to compaction prior to shear. The drained modulus numbers were estimated to be 110, 220, and 440 for relative compactions of 85, 90, and 95%, respectively. The corresponding drained modulus exponents are 0.25, 0.26, and 0.40, respectively. The drained modulus numbers are higher than those presented by Duncan et al. (1978) for soils that classify as a silty sand or clayey sand (SM-SC) according to the unified soil classification system. Duncan et al. (1978) suggest drained modulus numbers of 100, 150, and 200 for relative compactions of 85, 90, and 95%, respectively. Duncan et al. (1978) recommended a higher drained modulus exponent, 0.6, for these relative compactions.

The values of drained initial modulus of the normally consolidated specimens tested herein are lower than the values calculated from the modulus numbers and exponents reported by Boscardin et al. (1990) and Duncan et al. (1978) for effective confining pressures less than 300 kPa. The difference in drained initial modulus values is probably due to the effect of compaction-induced overconsolidation and the lack of back-pressure saturation on the hyperbolic parameters presented by Boscardin et al. (1990) and Duncan et al. (1978). Differences in soil type, lower compaction water contents, and higher dry unit weights used by Boscardin et al. (1990) and Duncan et al. (1978) also contribute to the difference in initial modulus. The drained modulus exponent of the kaolinite-silt mixtures is equal to unity while the modulus exponents reported by Boscardin et al. (1990) and Duncan et al. (1978) are significantly less than unity. Modulus exponents less than unity indicate that the specimens are overconsolidated. It should also be noted that the modulus numbers reported by Boscardin et al. (1990) and Duncan et al. (1978) are higher than those reported herein. A modulus exponent of unity and lower modulus numbers combine to yield values of initial modulus that are in agreement with previously reported values at effective confining pressures greater than 300 kPa. If desired, the parameters reported by Duncan et al. (1978) and Boscardin et al. (1990) can be used to augment the freshly deposited, normally consolidated hyperbolic parameters presented in Tables 3 and 5 to account for the effect of overconsolidation due to compaction and/or soil fabric at effective confining pressures less than 300 kPa. This will facilitate the estimation of hyperbolic stress-strain parameters for compacted and structured silts and clayey silts.

CONCLUSIONS

The main objectives of the present research were to characterize the drained and undrained stress-strain behavior of freshly deposited, normally consolidated silts and develop recommendations for hyperbolic stress-strain parameters for silts. To achieve these objectives, isotropically consolidated-drained and consolidated-undrained triaxial tests were conducted on silt mixtures with varying clay mineral contents. The percentages of clay mineral used in the silt mixtures are 0, 10, 20, 30, and 50% based on dry weight. Commercially available kaolinite and montmorillonite were mixed with the silt to investigate the effect of clay mineralogy. The effect of dry unit weight on the stress-strain behavior was investigated by compacting the triaxial specimens at standard Proctor relative compactions of 85, 90, 95, and 100%. The main conclusions concerning the stress-strain behavior and hyperbolic parameters of normally consolidated silts and clayey silts are summarized as follows:

1. The shear behavior of normally consolidated clayey silts is controlled by the percentage and mineralogy of the clay. At low clay mineral contents, silt exhibits dilative shear behavior even though the specimen is normally consolidated. At high clay mineral contents, silt exhibits a contractive shear behavior. The transition from dilative to contractive shear behavior is a function of the clay mineralogy and the percentage of clay mineral present based on dry unit weight. The shear behavior transition point occurred at a clay content between 15 and 20% for the kaolinite-silt mixtures and less than 10% for the montmorillonite-silt mixtures.
2. At clay contents less than 20%, the kaolinite-silt mixtures exhibited dilation under drained and undrained conditions even though the specimens were normally consolidated. Since the hyperbolic model does not account for dilation, the hyperbolic parameters presented for the low kaolinite mixtures should be used for soil-structure interaction analyses involving strains less than about 8%. At kaolinite contents greater than 20% and montmorillonite contents greater than approximately 5%, the shear behavior is contractive and the hyperbolic model provides an excellent representation of the measured stress-strain behavior for axial strains less than or equal to 20%.
3. The Mohr-Coulomb shear strength and hyperbolic stress-strain parameters can be estimated for freshly deposited, normally consolidated silts and clayey silts using the in situ dry unit weight, water content, applicable range of consolidation pressure, and the database presented herein. The maximum deviator stress and maximum pore-water pressure failure criteria were used to estimate the shear strength parameters from the drained and undrained triaxial tests, respectively.
4. Clay mineralogy, as well as the percentage of clay mineral, controls the shear behavior of silt. Increasing clay plasticity decreases the drained modulus number and shear strength parameters of the silt mixture. In addition, the plasticity decreases the percentage of clay required to reach the transition point between dilative and contractive shear behavior.

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