Highway Embankment on Soft Soils Case Study and Lessons Learned

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ABSTRACT

This case study describes failure of an interstate connecting-ramp embankment during construction and investigates performance of the prefabricated vertical drains (PVDs) installed to accelerate consolidation of the weak embankment foundation soils. The weak, fine-grained foundation soil experienced less drainage, and thus less consolidation and strength gain, than expected via the PVDs because of an overestimate of the design horizontal coefficient of consolidation. The inverse analyses show the failure was caused by lower than expected shear strength of the foundation soils and an over estimate of the compacted embankment shear strength. As a result, recommendations to estimate horizontal consolidation properties for PVD design and embankment shear strength parameters or future projects are presented.

INTRODUCTION

This paper presents a case study analyzing the failure of a 91 m (300 ft) long section of a connecting-ramp embankment (Ramp ES) between westbound Interstate-76 (I-76) to southbound Interstate-71 (I-71) in Medina County, Ohio. The 91 m (300 ft) long section between Stations 202+00 and 205+00 failed during construction and at just over 43% of the design height (9.2 m; 30 ft). The widening and reconstruction of I-71 at the I-76 interchange included the addition of a third lane for 5.2 km (3.2 miles) of I-71, construction of new ramps and embankments, and demolition and re-construction of fourteen (14) bridges. The plans and specifications for this project were completed in 2004.

A key design feature of the Ramp ES design is the use of prefabricated vertical drains (PVDs) to accelerate consolidation and increase shear strength of the weak, fine-grained foundation soils under the connecting-ramp embankment. The design of the PVDs sought to increase the foundation soil shear strength by a factor of 2.0 to 2.5 to develop an adequate factor of safety during placement, which is discussed below. Placement of 2.5 m of fill corresponds to an increase in applied stress of about 54.9 kPa based on a unit weight of 22.0 kN/m^3., which is in good agreement with the increase in pore-water pressure from 55 to 95 kPa. This means that the majority of the applied stress is initially carried by the pore-water pressure.

However, after only 2.4 m (8 ft) of embankment fill placement, or just over one-quarter of the full embankment height of 9.2 m (30 ft) at this location, tension cracks began to appear along the crest of the embankment. After the embankment height reached about 43% (4.0 m or 13 ft) of the full height (9.2 m or 30 ft), a 91 m (300 ft) long section of the embankment failed in 2007.

This paper presents an evaluation of the failure mechanism, PVD design and performance, embankment shear strength parameters and tension crack, and slope stability and bearing capacity analyses to facilitate future highway embankment design and construction in Ohio.

SUBSURFACE CONDITIONS AT RAMP ES

Figure 1 shows the embankment cross-section in the area of the slope failure, i.e., between Stations 202+00 and 205+00. The groundwater was found after drilling at a depth of about 1 m as shown in **Figure 1.** The soil profile generally consists of a thin (~0.3 m or 1 ft) layer of topsoil overlying various layers of fine-grained soil. The fine-grained soils are described as brown and/or gray, mottled, clayey silt or silty clay with lesser percentages of sand and gravel. Very soft to soft silty clays, sandy silts, and organic clays ranging from depths of 0.3 to 11.6 m (1 to 38 ft) had organic contents ranging from 3 to 84% as determined by ignition tests (ASTM D2974-14). The majority of this material classifies as an organic clay, not peat under ASTM D4427-13, because peat classification requires an organic content greater than 75% under ASTM D2974-14.

Table 1 presents the slope stability analysis input parameters for the subsurface cross-section in the failure area at Station 203+58 (see **Figure 1**) used for the inverse analyses conducted herein. To simulate the original embankment design, a singular value of undrained strength for each layer is used herein instead of estimating the undrained strength from an undrained strength ratio. As discussed below, the increase in effective vertical stress, and thus shear strength, caused by consolidation due to the embankment fill is the greatest at the embankment centerline and decrease towards the slope toe. As a result, the increase in shear strength due to consolidation along full length of the failure surface in **Figure 1** is probably small**.**

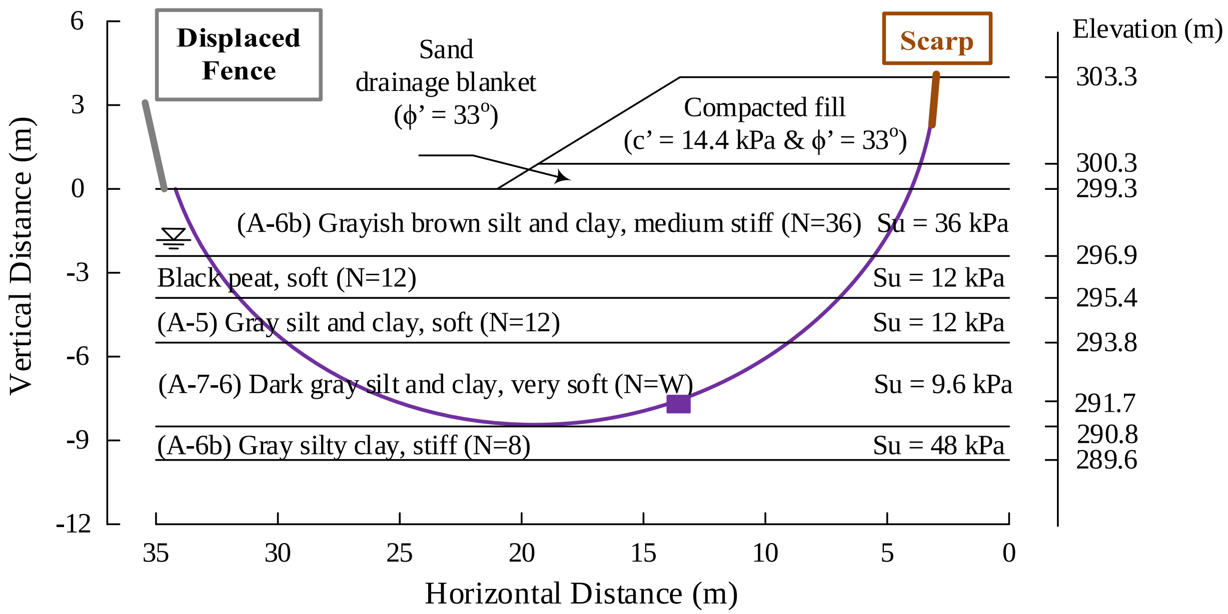

Figure 1. Subsurface cross-section for failed embankment, estimated undrained shear strengths, depth of movement at 12.5 m (~41 ft) in slope inclinometer at Station 200+00 (see solid block) at Elevation +291.7 m, and observed failure surface.

Table 1. Slope Stability Input Parameters for Subsurface cross-section at Station 203+58 based on Boring ES-8C.

Soil Type	Total and Saturated Unit Weights (kN/m^3/pcf)	Undrained Shear Strength (kPa/psf)	Effective and (ϕ') and Total (ϕ) Stress Friction Angles (degrees)
Compacted Fill	21.2/135	71.8/1500 & 14.4/300	ϕ'=33°
Sand drainage blanket	18.6/120	0	ϕ'=33°
(A6-b) gray/brown silt and clay	17.3/110	36/752	ϕ=0°
Black peat	11.8/75	12/250	ϕ=0°
(A-5) gray silt and clay	17.3/110	12/250	ϕ=0°
(A-7-6) dark gray silt and clay	15.7/100	9.6/200	ϕ=0°
(A-6b) gray silty clay	18.1/115	48/1000	ϕ=0°

RAMP ES DESIGN

The design of Ramp ES included a number of features to allow the weak foundation soils to consolidate and gain strength over time, e.g., PVDs and pore-water pressure monitoring, to achieve the desired long-term factors of safety for the embankment. Embankment fill placement could proceed until the measured pore-water pressures increased to 48.3 kPa (7 psi) above the baseline pore-water pressure of about 54 kPa (7.8 psi). If the change in pore-water pressure exceeded the baseline pressure plus 48.3 kPa (7 psi), i.e., about 102.0 kPa (14.8 psi), the fill placement had to cease until the pore-water pressure receded below 102.0 kPa (14.8 psi). The 48.3 kPa (7 psi) change in pore-water pressure was based on preliminary stability analyses and the PVDs being able to dissipate fill placement induced pore-water pressures. **Figure 2** shows the as-built fill height and piezometer readings versus time at Station 204+00, which is in the slope failure area between Stations 202+00 to 205+00. As expected, the pore-water pressures increased with increasing fill height but the increases were somewhat erratic. This may be due to non-uniform fill placement, which occurred due to some fill being concrete roadway rubble.

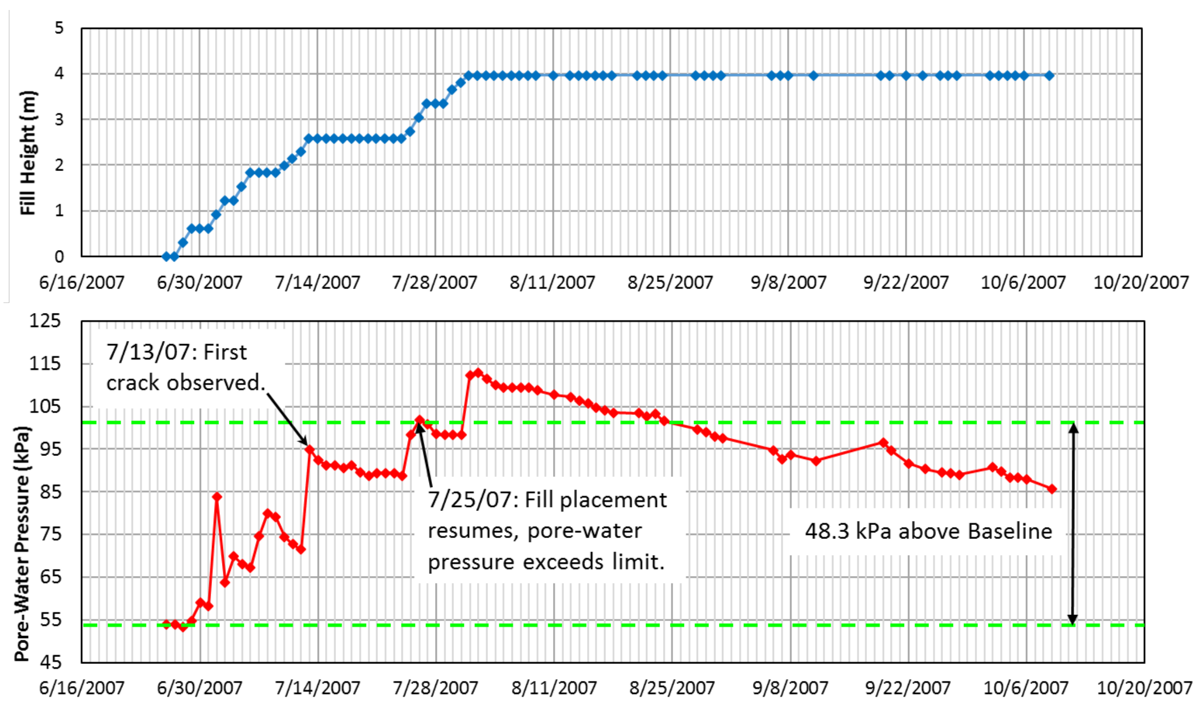

Figure 2. Fill height, baseline pore-water pressure, and pore-water pressure vs. time at Station 204+00 at Elevation 295.7 m (see **Figure 1**).

RAMP ES CONSTRUCTION FROM STA 202+00 TO STA 205+00

On 13 July 2007, when the embankment had reached only 2.4 m (8 ft) or just over one-quarter of the full embankment height (9.2 m or 30 ft) at this location, inspectors noticed tension cracks along the top of the embankment from Station 202+00 to Station 205+00. At that time, the foundation pore-water pressures had only increased 41 kPa (6 psi) above the 54 kPa (7.8 psi) baseline pore-water pressure, which is below the limiting increase of 48.3 kPa (7 psi) so construction could continue under the contract requirements.

A failure surface that corresponds to the tension cracks on the embankment crest, the inclinometer shear displacement at a depth of approximately 12.5 m (~41 ft) at Station 200+00, and the observed toe of the slide mass is shown in **Figure 1**.

Between 13 July 2007 and 24 July 2007, embankment fill placement ceased while a geotechnical evaluation was completed by ODOT. On 24 July 2007, ODOT decided that embankment fill placement could resume but recommended that survey (PK) nails be installed along the embankment toe to monitor rotational movement. Between 24 July 2007 and 1 August 2007, the surveyor (PK) nails installed along the embankment toe showed uplift in the failure area, which was probably initial evidence of rotational movement.

By 1 August 2007, the embankment work reached a height of 4.0 m (13 ft). However, on 1 August 2007 additional tension cracks were noticed in the same location as the 13 July 2007 cracks along the embankment crest. Also on 1 August 2007, the pore-water pressure in the foundation soils exceeded 110 kPa (16 psi), which is 56.5 kPa (8.2 psi) above the baseline pressure (54 kPa/7.8 psi)) or 8.2 kPa (1.2 psi) above the allowable pore-water pressure increase of 48.3 kPa (7 psi).

On 2 August 2007, embankment construction was stopped and ODOT began re-assessing the embankment stability and developing alternatives for the planned embankment construction. Between 2 and 6 August 2007 embankment tension cracking along the crest progressed. Finally, on 6 August 2007, the northern half of the embankment rotated vertically downward 0.46 m (1.5 ft) and the foundation soils near the northern toe of the slope raised up. Displacement of a nearby right-of-way fence (see **Figure 1**) indicated a deep-seated rotational foundation movement had occurred. Throughout embankment construction, there was little evidence of flow from the PVDs or the sand blanket. However, on 6 August 2007, water was observed draining from the sand blanket even though it had not been observed previously.

EVALUATION OF PREFABRICATED VERTICAL DRAIN PERFORMANCE

Consolidation of the weak foundation soils during embankment loading would require a significant amount of time because of the long vertical drainage path. As a result, one of the features of the Ramp ES design is the use of PVDs to accelerate primary consolidation of the weak fine-grained layers by reducing the drainage path and, in most cases, taking advantage of the larger coefficient of horizontal consolidation instead of the vertical coefficient (Mesri and Lo, 1991). The Ramp ES design utilized more than 99,060 m (325,000 feet) of PVDs installed on a 1.8 m (6 ft) equilateral triangle spacing, with an effective drainage area of approximately 2.9 m^2 (31 ft^2). The PVDs are 102 mm (4 in) wide and 6 mm (0.25 in) thick with a discharge capacity through the core of at least 1.9 liters per minute (0.5 gallons per minute) measured under a normal stress of 239.4 kPa (5,000 psf) after a period of 24 hours using a gradient of unity. The PVDs were installed with an anchor plate to keep the drain at or near the required depth when the T-shaped mandrel was removed during installation. The PVDs were installed to depths that fully penetrated the weak foundation layers and continued into the underlying firm soil.

Based on the measured pore-water pressures in **Figure 2**, the PVDs probably did not produce the expected strength gain because of:

1. an overestimate of the horizontal coefficient of consolidation, i.e., horizontal hydraulic conductivity, of the foundation soils,
2. filter cake formation on the filter geotextile surrounding the PVD that limited pore-water pressure dissipation,
3. clogging of the PVD due to filter geotextile incompatibility with the foundation soils, and/or
4. possible damage of the PVDs during installation as discussed below.

Mobilized Coefficient of Consolidation

The value of horizontal coefficient of consolidation, C_h, used to initially design the PVDs is 18.6 m^2/year (200 ft^2/year). This design value of C_h can be verified during fill placement and subsequent settlement using the Asaoka (1978) method to determine if consolidation and strength gain are occurring as expected and facilitate use of the Observational Method described by Peck (1969). The mobilized value of C_h of the weak foundation soils was estimated herein using the Asaoka (1978) method, which uses field settlements with time to estimate the mobilized value of C_h. This method works by: (1) selecting a series of measured settlements (s_1, s_2, s_3,…s_j, s_{j+1},…) corresponding to various times (t_1, t_2, t_3,…t_j, t_{j+1},…) but at a constant time interval, i.e., $t_{j+1} - t_j$ = constant; (2) plotting s_{j+1} against s_j; (3) drawing a straight line through the settlement data; and (4) extrapolating the line to intersect a 45° line through the origin (Mesri and Huvaj-Sarihan, 2009).

Urzua et al. (2016) show that the Asaoka (1978) method yields good estimates of the mobilized C_h when PVDs are used and horizontal drainage controls the consolidation process. When vertical drainage controls the consolidation process, the Asaoka (1978) method yields reasonable values of mobilized vertical coefficient of consolidation, C_v, and settlement when the degree of consolidation is greater than 50% (Urzua et al., 2016). However, if the degree of consolidation is less than or equal to 50% and vertical drainage controls the consolidation process, the Asaoka (1978) method yields unsafe (C_v value is too high) and another analysis method should be utilized (Urzua et al., 2016).

Table 2 presents the settlement data over time at Station 204+00 that is used below to estimate the mobilized value of C_h in **Table 3** using the procedure in Asaoka (1978). The point of intersection between the 45° line and the settlement data defines the EOP settlement and the slope of the line is used to estimate the mobilized C_h.

Table 2. Field settlement data over time at Station 204+00.

Date	Settlement (m/ft)	Settlement (m/ft)
	S (j)	S (j+1)
6/6/2007	0	0
6/27/2007	0.015/0.05	0
7/9/2007	0.159/0.52	0
7/17/2007	0.281/0.92	0
7/24/2007	0.287/0.94	0.476/1.56
8/1/2007	0.476/1.56	0.570/1.87
8/8/2007	0.570/1.87	0.625/2.05
8/15/2007	0.625/2.05	0.705/2.31
8/28/2007	0.705/2.31	0.750/2.46
9/12/2007	0.781/2.56	0.781/2.7
9/19/2007	0.824/2.70	0.824/2.86
9/29/2007	0.872/2.86	N/A

Table 3. Values of mobilized horizontal coefficient of consolidation (C_h) estimated using Asaoka (1978) method.

Station	**Mobilized C_h (m^2/year)**
202+00	7.7
203+00	8.4
204+00	6.4
205+00	19.0

The C_h values shown in **Table 3** at Stations 202+00 to 204+00 (6.4 to 8.4 m^2/year) are well below the value used in the initial design of 18.6 m^2/year (200 ft^2/year). Station 204+00, the center of the slope failure, produced the lowest value of mobilized C_h, i.e., 6.4 m^2/year (68.6 ft^2/year). This provides an explanation for the foundation soils not gaining sufficient undrained shear strength to maintain embankment stability. If the Asaoka (1978) method is used during construction and it is determined the consolidation and strength gain is not occurring as anticipated, the filling process can be modified to maintain embankment stability, which is in agreement with the Observational Method (Peck, 1969).

During the initial subsurface investigation, one consolidation test was performed on a gray silt and clay specimen that exhibited a liquid and plastic limit of 34 and 23, respectively. The results of this test yielded values of C_v of 1.78 and 2.47 m^2/year at effective stresses of 223 and 418 kPa, respectively. Typically the ratio of C_h to C_v ranges from 1.5 to 4 so the range of C_h from the laboratory values of C_v presented above is 2.7 to 9.9 m^2/year. This range of C_h is well below the design value of 18.6 m^2/year (200 ft^2/year).

Interestingly, the Asaoka (1978) method yielded a mobilized value of C_h of 6.4 m^2/year (68.6 ft^2/year) at Station 204+00, which yields a ratio of C_h to C_v of 3.6 to 2.6 using the laboratory consolidation test values of C_v. of 1.78 and 2.47 m^2/year, respectively. In other words, the Asaoka (1978) method yielded a ratio of C_h to C_v of 2.6 to 3.6, which is in excellent agreement with the typical ratio of C_h to C_v of 1.5 to 4.0. In hindsight, a ratio of C_h to C_v of 1.5 to 4 could have been used with the laboratory values of C_v to develop a reasonable design range of C_h for the PVDs (2.7 to 9.9 m^2/year) instead of assuming a value of 18.6 m^2/year (200 ft^2/year).

The ratio of C_h to C_v of 1.5 to 4 also can be used to verify the design value of C_h using the laboratory values of C_v. For example, the design value of C_h is 18.6 m^2/year (200 ft^2/year) corresponds to a ratio of C_h to C_v of 10.4 and 7.5 using the laboratory values of C_v of 1.78 and 2.47 m^2/year, respectively. A range of C_h to C_v ratio of 10.4 to 7.5 is not within the typical range of 1.5 to 4, which should have been an indication that the design value of C_h was too optimistic and consolidation and undrained strength gain would not occur as fast as designed.

In summary, the value of C_h used for design of PVDs should be in agreement with the typical range of the ratio of C_h to C_v 1.5 to 4. The Asaoka (1978) method can be used to assess the mobilized value of C_h during filling and consolidation so the fill process can be modified as needed to maintain embankment stability using the Observational Method described by Peck (1969).

Variances between estimated and mobilized values of C_h can result from soil type and the presence of a smear zone around the PVD, which is a cylinder of disturbed soil around the drain that is caused by the mandrel during PVD installation. The disturbed soil in the smear zone usually has a lower hydraulic conductivity and lower C_h than outside the smear zone, which results in a longer time to reach the end of primary consolidation than predicted. However, Mesri and Lo (1991) conclude that discharge capacity has a greater influence on degree of consolidation than the radius of the soil cylinder and smear zone. Drains of poor quality or drains that are damaged during installation can slow consolidation more than the smear effect (Mesri and Lo, 1991) so PVD discharge capacity should be maximized during PVD selection.

Degree of Consolidation

Figure 3 shows the predicted increase in degree of consolidation with time for the C_h estimated using the Asaoka (1978) method at Station 204+00 (6.4 m^2/year) and the initial design of value of

18.6 m^2/year (200 ft^2/year). Failure of the Ramp ES embankment occurred approximately 1.5 months after placement of the embankment fill began, which corresponds to about 86 percent consolidation if C_h had been 18.6 m^2/year (200 ft^2/year) (see **Figure 3**). If C_h had been 18.6 m^2/year (200 ft^2/year), the undrained strength would have been sufficient to prevent embankment failure. However, using a time of 1.5 months and the mobilized value of C_h of 6.4 m^2/year (68.6 ft^2/year) from the Asaoka (1978) method at Station 204+00 yields a degree of consolidation of less than 50 percent consolidation, which helps explain the slope failure and low mobilized undrained shear strength of the foundation soils at the time of failure.

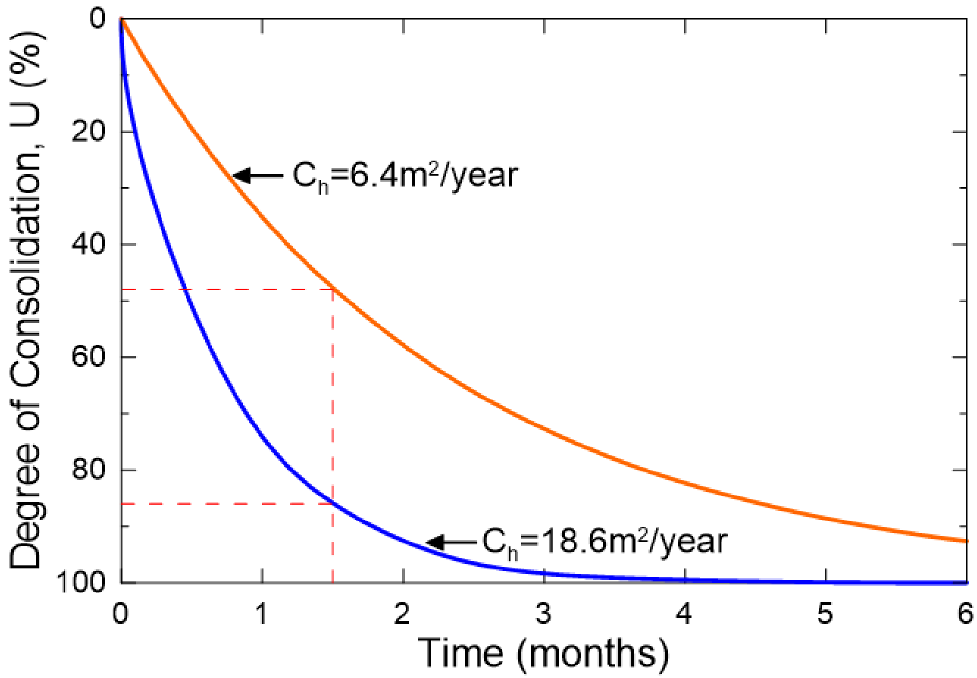

Figure 3. Degree of Consolidation versus Time for Station 204+00 for the design and mobilized values of C_h.

EVALUATION OF COMPACTED FILL STRENGTH

As mentioned above, the two main causes of the slope failure are: (1) lower than expected shear strength of the foundation soils and (2) an over estimate of the compacted embankment shear strength. This section investigates the shear strength of the compacted embankment fill.

The embankment strength parameters used in the initial design analysis are a total stress cohesion (c) or undrained shear strength of 71.8 kPa (1,500 psf) and a total stress friction angle (ϕ) of zero. This is in accordance with GB-6 (ODOT, 2010), which recommends using values of c of 71.8 to 95.8 kPa (1,500 to 2,000 psf) and ϕ of zero for the short-term analyses. The calculated FS using a c of 71.8 kPa (1,500 psf) and a fill height of 4.6 m (15 ft) is 1.64, which confirms the original design met ODOT FS requirements, but over-predicted the field FS because the slope failed, i.e., FS~1.0.

If an undrained shear strength is used to model a stiff compacted fill over soft foundation soils, a tension crack should be included in the embankment. A tension crack is required because of the strain incompatibility of the stiff embankment and the soft foundation soils. This results in the percentage of strength mobilized in the embankment being smaller than in the foundation soils (Chirapuntu and Duncan, 1976). This incompatibility results in the tensile stresses developing in the embankment due to the lateral deformation of the soft foundation soils. This results in the development of a tension crack in the embankment. The depth of this tension crack, H_{crack}, can be estimated assuming a planar failure surface and force equilibrium using the following expression:

$$\mathrm{H}_{crack} = \frac{2 * c_{fill}}{\gamma_{Fill} * \tan(45^0 - \frac{\phi_{fill}}{2})} \qquad (1)$$

where γ_{fill}, ϕ_{fill}, and c_{fill} are the unit weight, total stress friction angle, and total stress cohesion of the compacted fill. For an undrained condition, the value of ϕ_{fill} is set to zero so Equation (1) reduces to:

$$\mathrm{H}_{crack} = \frac{2 * c_{fill}}{\gamma_{Fill}} \qquad (2)$$

Equation (2) and a value of c_{fill} of 71.8 kPa (1,500 psf) results in a tension crack depth of 6.7 m. Equation (3) and a value of c_{fill} of 95.8 kPa (2,000 psf) results in a tension crack depth of 9.0 so the range in tension crack depth is 6.7 to 9.0 m for the range of undrained strength recommended by GB-6. Both of these crack depths exceed the height of the embankment when the Ramp ES slope failure started at a fill height of 4.0 m (13 ft). As a result, no shear resistance should have been used for the embankment in the design stability analyses for Ramp ES because an undrained shear strength was being used to model the compacted fill strength.

$$\mathrm{H}_{crack} = \frac{2 * c_{fill}}{\gamma_{Fill}} = \frac{2 * (71.8\,\mathrm{kPa})}{21.2 \frac{kN}{m^3}} = 6.7m \qquad (3)$$

The critical factor of safety using a circular failure surface search, shear strengths shown in **Figure 1,** no tension crack, and one-half of the final embankment height or 4.6 m (15 ft) is 1.59, which indicates stability. However, this limit equilibrium analysis showed significant tension in the upper portion of the embankment, i.e., negative normal stresses on the base of several of the vertical slices in the embankment.

Performing the same analysis with a tension crack for the full embankment depth, i.e., 4.6 m (15 ft) as suggested by Equation (3), yields a critical factor of safety of 0.91. Therefore, if an appropriate tension crack was included in the initial design stability analysis, failure would have been predicted. The embankment failed at a height of only 4.0 m (13 ft) so the same analysis as above was performed with a tension crack the full height of the constructed embankment, i.e., 4.0 m (13 ft), and the critical factor of safety is 1.02, which is in excellent agreement with the slope failure occurring at an embankment height of 4.0 m (13 ft).

SUMMARY AND RECOMMENDATIONS

Based on the post-failure PVD and slope stability analyses performed for this interstate connecting-ramp embankment, the following lessons learned or recommendations are made for evaluating the stability of stiff, compacted embankments over soft foundation soils:

- A reasonable range of the ratio of horizontal to vertical coefficient of consolidation is 1.5 to 4.0. This empirical range can be used to guide selection of the initial value of horizontal coefficient of consolidation for PVD design and/or to confirm laboratory data.
- The Asaoka (1978) method can be used during embankment construction and settlement to estimate the mobilized value of horizontal coefficient of consolidation for comparison with the design value to verify that consolidation and strength gain are occurring as predicted. If consolidation is not occurring as fast as predicted, then fill placement can be modified to maintain stability, which is in agreement with the Observational Method proposed by Peck (1969).
- If the stiff, compacted embankment overlying soft foundation soils is modeled with an undrained shear strength and a friction angle of zero in stability analyses, a tension crack must be included in the stability analysis otherwise the strength of the embankment will be overestimated. The depth of the tension crack can be estimated using force equilibrium and a planar failure surface derived expression in Equation (1) or the empirical expression developed by Chirapuntu and Duncan (1976).

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