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# SIMULATING THE LOADING ENVIRONMENT AT A RAILWAY BRIDGE TRANSITION ZONE

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ABSTRACT
This paper illustrates the progressive settlement of a railway bridge approach transition zone using a three-dimensional dynamic numerical model which includes the train truck, rails, ties, ballast, subgrade, and bridge structure. Assuming newly laid and compact ballast, a settlement model that relates tie load to ballast settlement is implemented in an iterative fashion to evaluate the bridge approach response for 28 MGT. The results show: (1) the development of tie-ballast gaps that progressively increase in size and expands outwards from the bridge abutment, (2) a redistribution of load outwards from the bridge abutment as the tie-ballast gaps develop, and (3) a ballast surface profile that attempts to minimize wheel and tie loads by distributing the wheel load amongst all the ties. These results assume perfectly homogeneous ballast conditions, which is not always true as ballast conditions, such as degradation, confinement, moisture, and fouling, may spatially vary within the approach. To test the effects of heterogeneity, a sensitivity analysis randomly varies the settlement by +/- 0.5 mm. This slight variation can result in increased loads up to 80%. This signifies that uneven ballast settlement from heterogeneous ballast conditions may play a significant role in increasing loads within transition zones.

## INTRODUCTION

The expensive and reoccurring problem of differential movement, i.e. transient and permanent vertical displacements, at railway and highway bridge transition zones is a problem across the world (Stark et al., 1995; Long et al., 1999; Nicks, 2009; Coelho, 2011; Stark and Wilk, 2016). Typically the approach substructure settles at a greater rate than the surrounding track resulting in rail profile deviation, e.g. dips in rail elevation, within the transition zone (Stark and Wilk, 2016). In railroads, this represents a safety concern and the problem is especially important because of the expansion and upgrade of high-speed passenger rail, which is more sensitive to sudden changes in rail elevation. The causes of differential settlement at railroad bridge transition zones vary by site but can generally be attributed to (1) lack of track settlement on the bridge, (2) increased loading in the approach, and (3) reduced-performance ballast/subgrade conditions in the approach.

Numerical modeling techniques are commonly used to determine the cause and magnitude of increased loads in railroad bridge approach transition zones (Nicks, 2009; Wang et al., 2015; Stark et al., 2015). Typically, these models investigate how differential stiffness between the bridge and approach produces an increased wheel load on the bridge or in the approach. Results show minor increases in wheel load of about 10 to 20% (Plotkin and Davis, 2008; Nicks, 2009; Wang et al., 2015; Wilk, 2017). More recent studies have shown differential settlement between the bridge and approach or within the approach produce increased loads of over 50% (Nicks, 2009; Wang et al., 2015; Wilk, 2017), suggesting that settlement plays a larger role than stiffness. However, these simulations require an assumed ballast surface profile that may not be representative of physical track. Therefore, analyses that simulate the progressive settlement over time have recently been implemented (Wilk and Stark, 2016; Wang and Markine, 2016).

This paper simulates the progressive settlement of a railroad bridge approach transition zone and investigates how homogeneous and heterogeneous ballast assumptions affect the loading environment

within the bridge approach. The simulation uses an iterative procedure to simulate the change in settlement and loading with increased loading cycles.

## NUMERICAL MODEL

The three-dimensional finite element software LS-DYNA, which specializes in non-linear transient dynamic behaviour, was selected to simulate the progressive settlement of a bridge approach transition zone. The bridge approach finite element mesh is displayed in Figure 1 and is based off the Upland Street Bridge near Chester, Pennsylvania because it is the site of previous site instrumentation (Stark and Wilk, 2016). The simulation consists of a cart representing the secondary suspension system of a train, 136-RE rail, concrete ties, and a five layer substructure that represents the physical substructure of an instrumented site. The bridge includes a masonry wall, an open deck bridge with timber ties on the bridge, and W-beams underneath the bridge. The stiffness of the bridge is greater than the approach track which is expected to produce impact loads when the front wheels of the cart pass onto the bridge abutment.

The cart is modeled to represent the secondary suspension system of the Acela high-speed passenger power car that passes over the Upland Street Bridge. It consists of four wheels with the axles spaced 2.8 m (9.33 ft.) apart to replicate the first bogie of a single Amtrak Acela power car. The cart mass is contained in the cart center with a density such that each wheel applies a static wheel load of 100 kN. The axles and cart mass are connected with four sets of vertical and horizontal springs and vertical dampers. The values of the vertical and horizontal springs are 7.3e5 N/m and 2.2e9 N/m and damper values are set to 7.3e6 N*s/m. The velocity of the cart is inputted as 177 km/hr (110 mph) to replicate the operating speed of the high speed trains at the instrumented site.

The rail geometry is modeled after 136-RE rail with the density, Young's Modulus, and Poisson's Ratio representing the steel in a 136-RE rail, which are 7.85 g/cm$^3$, 200 GPa, and 0.28, respectively. The concrete ties have a spacing of 0.6 m (2-ft), a width of 0.23 m (0.75-ft), and density, Young's Modulus, and Poisson's Ratio matching the values of concrete, which are 2.97 g/cm$^3$, 21 GPa, and 0.15 respectively. Table 1 presents the sublayer thicknesses and modulus values for each layer estimated using an inverse analysis to match field and numerical data (Wilk, 2017). All substructure materials have a density of 3.85 g/cm$^3$ and Poisson's Ratio of 0.4.

**Table 1: Substructure layer thickness and moduli used in both the open track and bridge approach numerical models.**

| | | Layer 1 | Layer 2 | Layer 3 | Layer 4 | Layer 5 |
|---|---|---|---|---|---|---|
| **Thickness** | **[mm]** | **305** | **127** | **508** | **721** | **849** |
| **Modulus** | **[MPa]** | **207** | **67** | **33** | **32** | **59** |

All elements are fully integrated and assume homogeneous, isotropic, linear elastic behaviour and all model boundaries have pinned and non-reflective boundary conditions. The non-reflective boundary conditions absorb pressure and shear waves, preventing the pressure and waves from reflecting back into the model. The distances to the boundaries are sufficient to prevent boundary effects from influencing the model.

To simplify the track modeling, the only assumed track defects are tie-ballast gaps, where the bottom of the tie and top of the ballast are simulated as separate entities with contact surfaces. This means all track non-linearity occurs from geometric discontinuities. The tie-ballast gap height is also assumed to be constant under a single tie. While this paper focuses on the tie-ballast gap, the mechanism of load redistribution can occur from any gap or defect within the track system, i.e. rail-fastener gap, fastener-tie gap, or from sudden changes in substructure modulus (Dahlberg, 2010). Also, unsupported or poorly

supported ties in physical track will likely have differential gaps and stiffness along the length of a single tie, which can influence load distribution under a single tie.

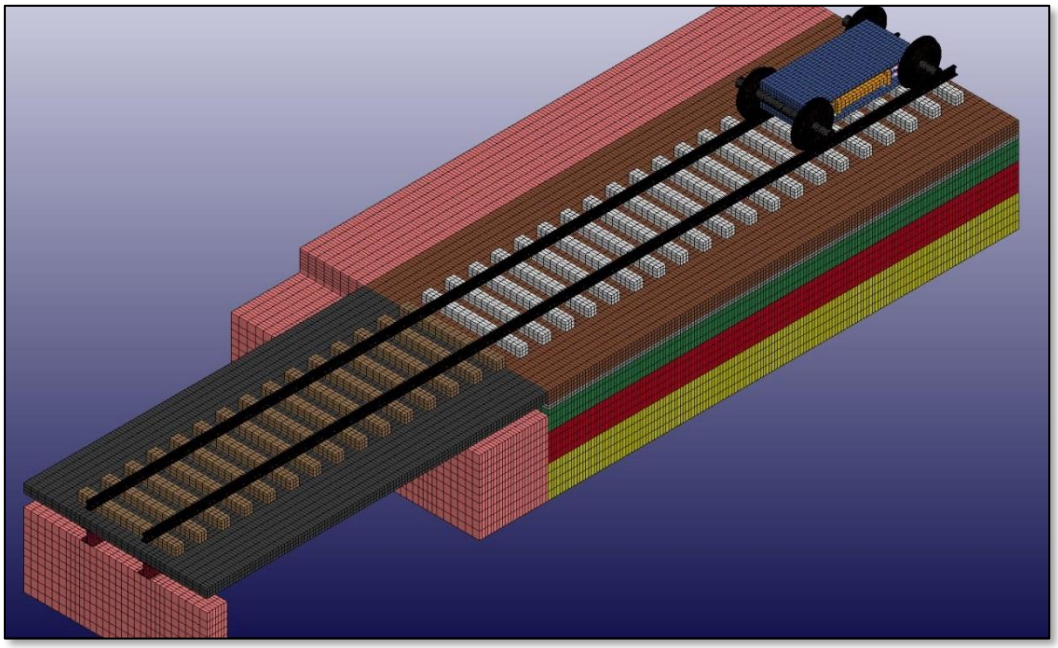


**Figure 1: LS-DYNA Finite Element Mesh showing Upland Street Bridge Approach Site with a Rolling Cart.**

The four model outputs from the finite element mesh are wheel-rail contact force, tie-ballast contact force, tie displacement, and ballast displacement. The wheel-rail and tie-ballast contact forces are calculated using master-slave penalty methods. This method checks for penetration of slave surfaces, i.e. top of rail surface, through the master surface, i.e. bottom of wheel surface, and applies a proportional force to resist the penetration. These forces are defined as wheel-rail and tie-ballast contact forces.

The tie-ballast contact force is of interest as it dictates the force that each tie transfers to the underlying ballast. In this paper, the tie-ballast contact force is normalized by the static wheel load (100 kN) and presented as the normalized tie load. Physically, this represents the percent of the static wheel load that gets transferred to the underlying tie. If all ties are well supported and are in intimate contact with the ballast, each tie will experience 40% of the static wheel load (normalized tie load = 40%). To determine the percent increase from this intimate contact situation, the percent increase or decrease from intimate contact is also presented. For example, if a tie receives a tie load of 60 kN, the normalized tie load equals 60% (60 kN / 100 kN = 60%) and a percent increase from intimate contact of 50% ((60% - 40%) / 40% = 50%).

## SETTLEMENT MODEL

The first step of developing a progressive settlement analysis is to select an appropriate settlement model to represent settlement with repeated loading. To calculate ballast settlement within each iteration step, the empirical settlement model proposed by Sato (1997) and modified by Dahlberg (2001) was implemented. This model is well suited for differential loading environments, such as transition zones, because the calculated ballast settlement (y) is only a function of the load applied at the tie-ballast interface (P), a primary output of the numerical model.

Empirical settlement models developed from laboratory ballast testing data is preferred over continuum plasticity laws built-in to existing numerical software (Indraratna et al. 2012) or discrete element modelling (DEM) (Tutumluer et al., 2013; Chen et al., 2015) because of greater flexibility with empirical models and the ability to produce settlement in discrete increments instead of requiring simulation of

every wheel pass. This is necessary to keep analyses in the timeframe of weeks or months as opposed to years or decades. Additionally, empirical settlement models that incorporate the number of loading cycles (N-value) (Dahlberg, 2001) were not considered for this particular analysis because track settlement in transition zones do not always follow the logarithmic settlement curve observed in controlled laboratory settings (Stark and Wilk, 2016) and those models typically assume a constant applied load and do not account for load variation with time. The implemented empirical settlement relation from Dahlberg (2001) is plotted in Figure 2 and the equation is the following:

$$y = 5.87E^{-9} * (P - 25)^4$$

where y is the ballast settlement in mm after 10,000 load cycles and P is the tie-ballast contact force in kN.

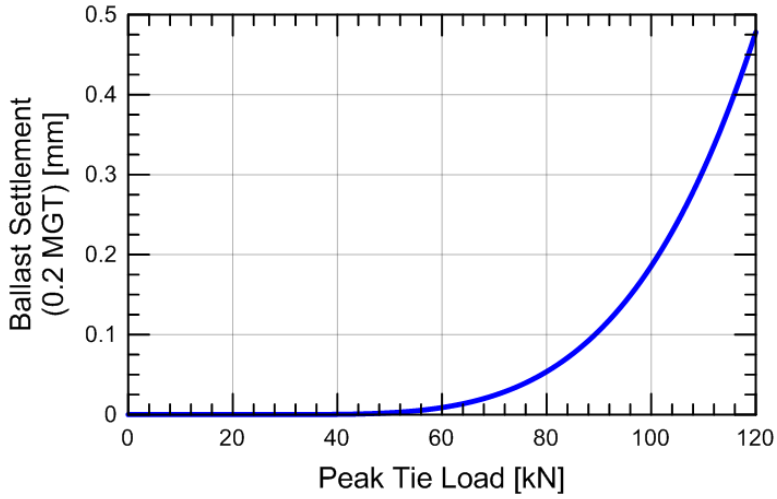



**Figure 2: Relation between peak tie load and ballast settlement from 10,000 load cycles for a 4th order settlement model.**

A simple settlement model that is only a function of tie load is considered suitable for the initial analyses because the goal of the numerical simulation is to conceptually investigate changes in loading environment from the progressive settlement of a transition zone and not replicating or predicting field behavior. Physically, ballast settlement is a complex process that involves particle rearrangement, lateral movement, and degradation and is dependent on numerous factors, including ballast density, gradation, moisture content, rock type, angularity, hardness, confinement, rotation of principal stresses, loading material (concrete v. timber), and impact (Indraratna et al. 2012b). These factors can later be incorporated into the settlement. The main assumptions included in the proposed settlement model are:

- The model assumes ballast settlement will not occur at tie loads less than 25 kN and a fourth-power relationship exists between ballast settlement (y) and tie-ballast contact force (P). This makes the model sensitive to higher loads than a linear relationship between ballast settlement and tie-ballast contact force. However, analyses with a linear relation do not show a significant difference in final results (Wilk, 2017).

- The settlement model does not account for ballast density, gradation, moisture content, rock type, angularity, hardness, confinement, rotation of principal stresses, etc., which has been shown to influence settlement magnitudes (Indraratna et al., 2012b). This means the model assumes a homogeneous ballast condition and will not represent: (1) spatially varying ballast conditions, (2) high-magnitude ballast settlement often observed directly after tamping, and (3) decrease in ballast settlement with increasing ballast density. These factors and the implications on model results are discussed in subsequent sections.

- A typical iteration applies 20,000 loading cycles, which is considered to be equivalent to about 0.4 MGT (million gross tons). This value assumes 200 kN (45 kip) axle loads or 100 kN (22.5 kips) wheel loads and is a reasonable assumption for high-speed passenger trains.

- The model assumes track settlement only occurs in the ballast layer and ballast settlement is homogeneous under the tie.

## ITERATIVE PROCEDURE

An iterative procedure is adopted to simulate the settlement of a bridge transition zone because settlement is not built into the numerical model and is expected to change with time. This means the geometry of the mesh is updated prior to every iteration. The four steps of the iteration procedure are described below:

1. A dynamic numerical simulation of the cart passing over the transition zone is completed with the model outputting the wheel loads, tie loads, tie displacement, and ballast displacements.

2. The peak tie loads from both the front and back axle of the cart are determined for Ties 1 through 10.

3. The ballast settlement under Ties 1 through 10 is calculated using the modified Dahlberg (2001) settlement model described above for both the front and back axles independently. The settlement values from each axle are then summed. The settlement of Ties 11 and greater are assumed to be equal to Tie 10 and represent open track.

4. The calculated settlements from Step 3 are added to the existing cumulative settlements under each tie and are incorporated into the numerical model geometry for the next iteration.

An important parameter in the iterative procedure is the representative MGT value of each iteration step, i.e. 0.4 MGT or 20,000 wheel passes. This is conceptually similar to a “time-step” and can significantly affect the simulation results because the transition zone loading environment is sensitive to local differential ballast settlements. This means large iteration steps can produce increased loads that could be avoided if using smaller iteration steps.

To show this, the maximum normalized tie load of a tie within a transition zone is determined using iteration steps of 0.2, 0.4, and 0.8 MGT. For 0.2 and 0.4 MGT iterative steps, the results are identical and the tie loading seems to be at a stable “equilibrium state”. However, the tie loads significantly deviate if assuming iteration steps of 0.8 MGT. This behavior indicates the progressive analysis has come out of “equilibrium” and entered a state in which the load fluctuates between adjacent ties every iteration. In this case, the load path at 4.0 MGT distributes load onto Tie 6 and away from adjacent Ties 5 and 7. This produces large settlement of the ballast underneath Tie 6. During the next iteration, the load path then shifts away from Tie 6 and onto adjacent Ties 5 and 7, resulting in large settlements under those ties. The process will continue indefinitely unless smaller iteration steps are introduced to allow the progressive analysis to re-enter a state of “equilibrium”. Ballast settlement technically occurs after every wheel pass (1 load pass), which is unrealistic to replicate from a computational time standpoint, so 0.4 MGT (20,000 load passes) increments is considered sufficient for this analysis.

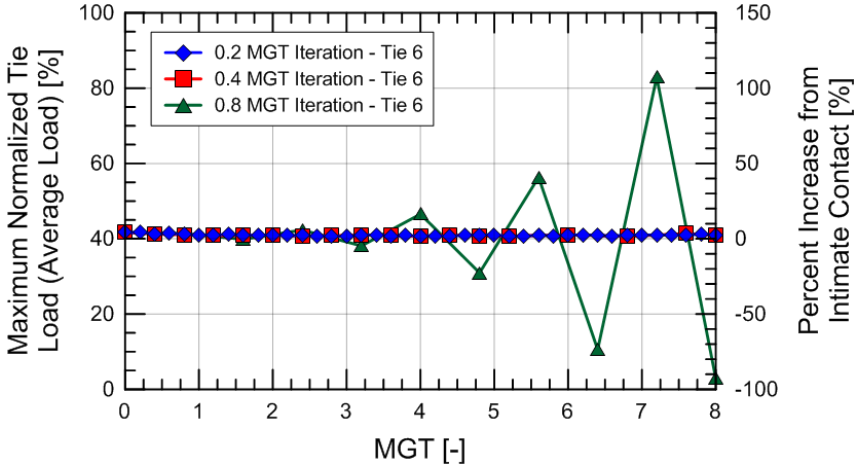



**Figure 3: Comparison of maximum normalized Tie 6 load (front and back wheel average) with iterations steps of 0.2, 0.4, and 0.8 MGT.**

PROGRESSIVE SETTLEMENT ANLAYSIS

The progressive settlement analysis was conducted at 0.4 MGT increments up to a total of 28 MGT. The analysis discontinued at 28 MGT because the assumption of Ties 11+ having identical loading and settlement as Tie 10 did not hold as the load shifts farther away from the bridge abutment with increasing settlement and load redistribution. An overview of the results is presented below.

First Iteration (0.0 MGT)

The first simulation of the cart passing over the bridge approach assumes newly laid and compacted ballast so no ballast settlement or tie-ballast gaps are present (Figure 4a). This results shows a 20% increase in the dynamic back wheel load from static conditions (120 kN) that occurs about 10 feet from the bridge abutment. This increased back wheel load is distributed primarily to Ties 5 and 6, which show increased loads of 20% and 7.5%, respectively (normalized tie loads of 47% and 43%) from the assumption of intimate tie-ballast contact (normalized tie load = 40%). The normalized tie load and percent increase from intimate contact of all ties can be referenced in Figure 4(b)

While a normalized tie load of 40% would produce a ballast settlement of 0.11 mm at 0.4 MGT (see Figure 2), the increased load at Tie 5 from the coupling of the front and back axles produces 0.18 mm of ballast settlement, almost doubling the “standard settlement”. This local differential settlement within the approach is a response to differential stiffness between the bridge and approach.

Second Iteration (0.4 MGT)

The ballast settlement from the initial run is incorporated in the numerical model for the second iteration analysis by decreasing vertical grid sizes of the ballast elements underneath each tie in the transition. The differential settlement within the transition zone is expected to change the wheel load distribution amongst the underlying ties and cause the load to shift from ties with the greatest ballast settlement to adjacent ties with lesser amounts of ballast settlement. This load redistribution mechanism is illustrated in Figure 4(b) with the second iteration analysis showing a reduction in load at Ties 5 and 6 and in increase in load at Ties 3, 4, and 7. For example, the normalized tie load at Tie 5 from the back wheel decreases from 47% (20% increase from intimate contact) to 44% (12% increase from intimate contact) and a similar reduction is observed for the front wheel. While this decrease in tie load may not seem significant, the non-linear tie load/ballast settlement relationship reduces the ballast settlement under Tie 5 from 0.18 mm between 0.0 to 0.4 MGT to 0.13 mm between 0.4 to 0.8 MGT (Figure 4c). This results in Tie 5 and 6 still experiencing the greatest dynamic loads and settlement but to a lesser degree than the 0.0 MGT analysis.

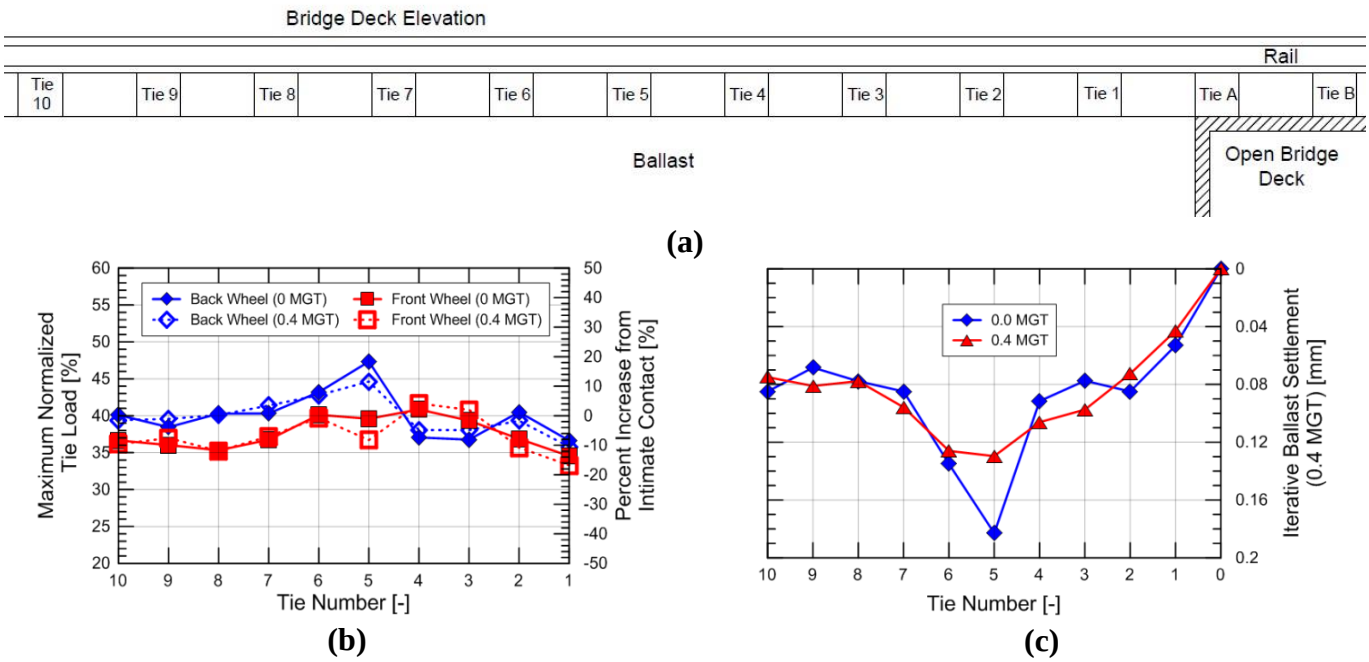



**Figure 4:** **(a) track profile during initial analysis, (b) Maximum normalized tie loads and resulting (c) iterative ballast settlement from train passes at 0.0 and 0.4 MGT.**

This suggests the ballast will attempt to settle in a manner that reduces tie loads within the transition zone and allow the transition zone to enter a stage of "equilibrium" in which tie loads are minimized. Therefore, any track experiencing increased loads from differential stiffness, i.e. pre-equilibrium, will subsequently experience differential ballast settlement that results in a better wheel load distribution amongst transition zone ties, i.e. equilibrium.

Long-Term Behaviour

To investigate the long-term settlement behavior of the transition zone, the cumulative ballast settlement profile, tie-ballast gaps, transient displacements, and tie loads are recorded with increasing MGT. The track profile at 28 MGT with proportional but exaggerated settlements is displayed in Figure 5(a) and illustrates how the rail hangs from the bridge deck and the development of tie-ballast gaps within the approach.

Figure 5(b) shows increasing cumulative ballast settlement with increasing MGT and the gradual shifting of maximum cumulative ballast settlement from under Tie 5 to Tie 7 during the duration of the analysis. This trend would be expected to continue as the load shifts farther from the bridge abutment. The maximum cumulative settlement at 28 MGT is 9.4 mm at Tie 7 and 8.9 mm at Tie 10, both of which are close but slightly greater than the predicted 7.6 mm for a normalized tie load of around 40%. Due to the differential settlement between the bridge deck and the transition zone, tie-ballast gaps develop in the transition zone and are shown in Figure 5(c). Initially, the tie-ballast gaps appear only under Ties 1 and 2 but gradually expand outwards and increase in magnitude as bridge and open track rail elevations continue to deviate.

The transient displacement from the cart passing over the transition zone is displayed in Figure 5(d). This shows a deviation between the settlement and transient displacement profiles where the transition zone (Ties 1 through 8) experience significantly greater displacements than the open track (Tie 10). This behavior agrees with the measured results at multiple transition zone locations (Stark and Wilk, 2016). The varying transient displacement in the transition zone is primarily explained by tie-ballast gap magnitudes because the ballast stiffness is assumed to be homogenous and loading is relatively similar across the transition zone.

The development of a tie-ballast gap has implications on track behavior because track system discontinuities allow for more movement and impacts between track components, and thus component degradation. For example, the freely moving tie will establish contact with the ballast during train loading and can result in increased tie wear and ballast degradation due to grinding and impact between the tie and ballast. Increased ballast settlement from the existence of tie-ballast gaps was observed in laboratory testing by Selig and Waters (1994) and increased tie accelerations at the moment of tie-ballast impact was observed in field testing by Wilk et al., (2016).

Figure 5(e) displays the normalized tie loads. A gradual increase in load is observed in Ties 6 through 10 while a gradual decrease in load is observed in Ties 1 through 4. The load experienced by Tie 5 remains essentially constant throughout the analysis excluding the initial run at 0.0 MGT. This load redistribution represents a shift of loading away from the bridge abutment as the ballast near the bridge settles resulting in unsupported ties. It is notable that the tie loads did not significantly increase as the differential settlement increased. This result differs from previous analyses with assumed ballast surface profiles (Wilk, 2017) and suggests the ballast surface profile from the progressive surface analysis is such to minimize tie loads.

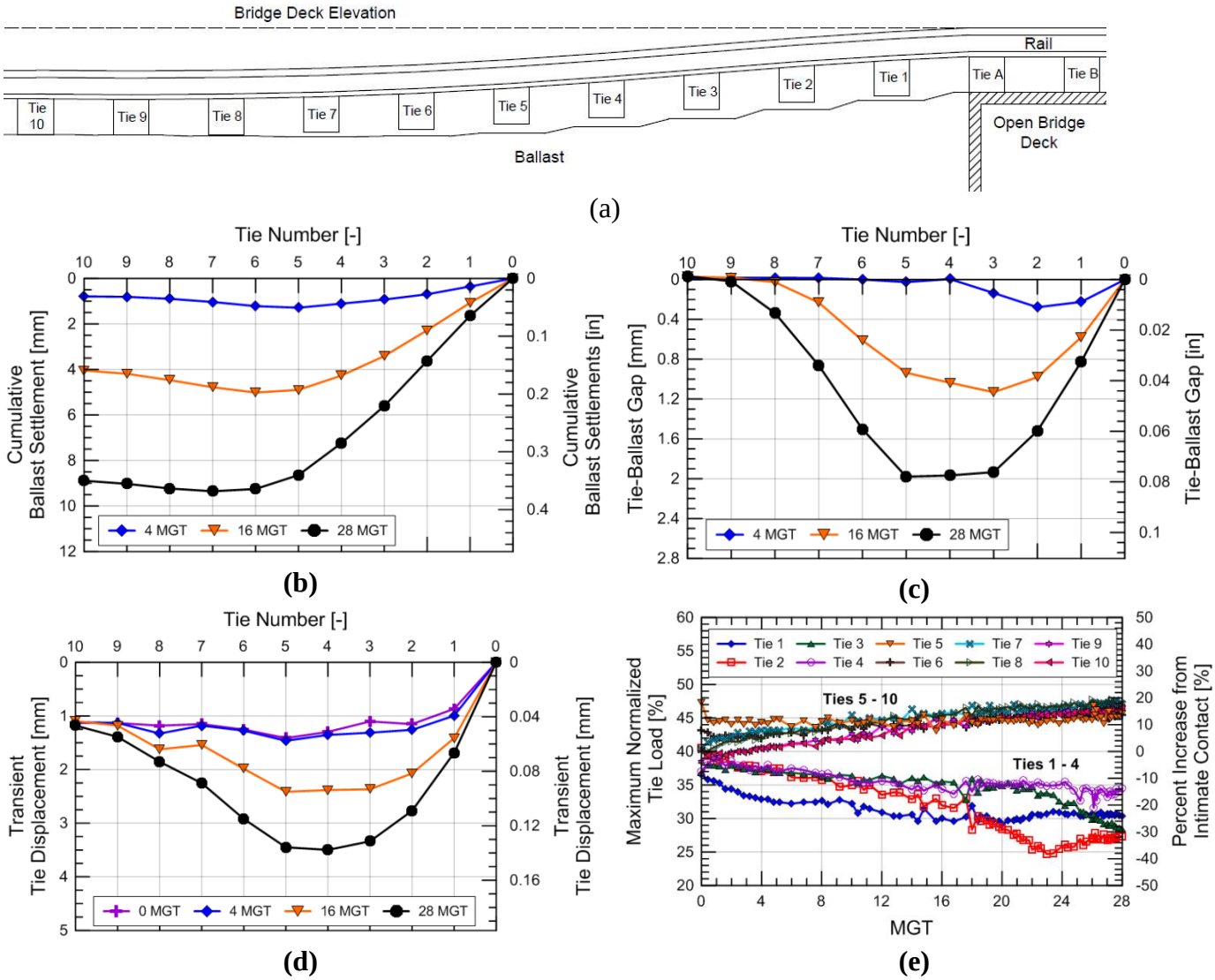



**Figure 5:** **(a) ballast settlement profile at 28.0 MGT and cumulative (b) ballast settlement, (c) tie-ballast gaps, (d) peak transient displacement, and (e) peak tie load with increasing MGT.**

Discussion

The results of the progressive settlement analysis provide insight to transition zone performance. As expected, the near rigid bridge deck represents a restricting condition that produces differential settlement between the bridge and approach, resulting in the gradual shifting of tie loads away from the bridge deck. Tie-ballast gaps develop near the entrance bridge abutment in reaction to the differential elevation between the rigid bridge and settling approach.

Comparisons between the results at 28 MGT and field observations show general agreement in behavior but the simulation does not replicate the field measured differential settlement between the transition zone and open track at poorly performing sites. For example, the difference between ballast settlement in the transition zone and open track in the numerical model (9.35 mm v. 8.89 mm) is less than some field measurements (14 mm v. 1 mm) (Stark and Wilk, 2016). This is not to say that some transition zones do not experience similar settlements in the approach and open track but that the transition zones of interest are typically sites displaying differential movement between the approach and open track. This suggests the numerical model is not simulating the increased load environment in the transition zone and/or the ballast will settle at greater rates than the predicted settlement relation in Figure 2. This will be addressed in detail below.

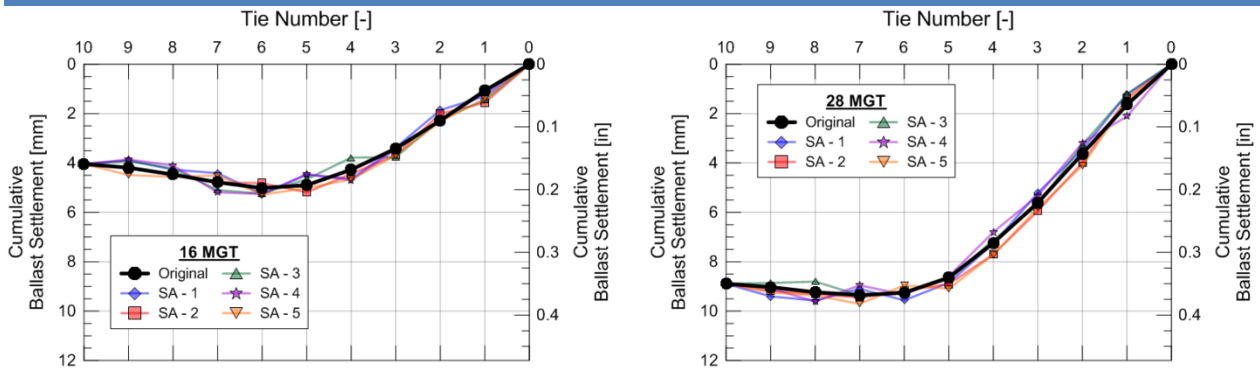
One of the primary observations from the progressive settlement analysis is that the track naturally attempts to find a state of "equilibrium" that evenly distributes the wheel load amongst the underlying ties. Essentially, if a single tie experiences a greater load than the surrounding ties, the ballast underneath that tie then experiences a greater settlement. During the following loading increment, the greater settlement under that tie reduces the load being distributed to that tie as the wheel load is shifted to the surrounding, better supported ties (Lundqvist and Dahlberg, 2005; Stark et al., 2015). This results in greater loads and settlement of the surrounding ties and lower loads and settlement at the tie of interest. If this process continues, the track should naturally settle in a manner that keeps it in "equilibrium". Analyses presented elsewhere investigate the effects of a linear settlement relation, initial settlement from tamping, and the inclusion of influence of tie-ballast gaps. These variations result in a different ballast surface profile, but the general concept of the track finding an "equilibrium state" holds true (Wilk, 2017).

One of the primary assumptions of model that results in this behavior is that the simulated ballast material is a homogenous material that will not experience changes in density, gradation, fouling content, moisture content, or confinement of time or distance. In reality, the ballast condition can vary from tie to tie, producing slightly different behaviour along the track. To test the effect of heterogeneous ballast, a sensitivity analysis is presented in the following section.

SENSITIVITY ANALYSIS

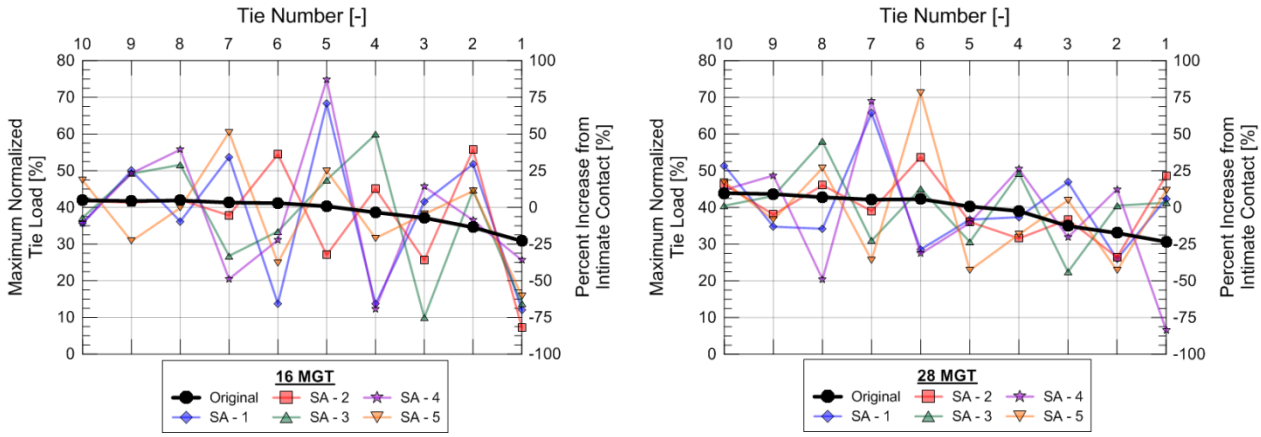
A sensitivity analysis is performed to investigate the effects of heterogeneous ballast in bridge approaches. To accomplish this, the ballast settlement is randomly varied within a range of +/- 0.5 mm at 16 and 28 MGT. For each situation (16 and 28 MGT), five analyses are conducted and results are presented in Figure 6 and 7.

The cumulative ballast settlement profiles inputted into the numerical models are displayed in Figure 6. As expected, the profiles show slight random variations in settlement at each tie from the original simulation displayed in the previous section. The loading in Figure 7, however, displays a wide range of tie loads with percent increased tie load from intimate contact exceeding 80%. This shows that slight settlement variations from the "equilibrium" state can significantly increase loads within the bridge approach and may be a potential explanation for the increased loading and settlement environment.




(a) (b)

**Figure 6: Ballast surface profile for sensitivity analysis (SA) for (a) 16 MGT and (b) 28 MGT.**




(a) (b)

**Figure 7: Normalized tie load for sensitivity analysis (SA) for (a) 16 MGT and (b) 28 MGT.**

SUMMARY

A numerical model was developed that simulates a train truck entering an open deck bridge and is based off the track geometry at Amtrak's Upland Street Bridge in Chester, PA. The objectives of the model are to simulate the progressive settlement of a railroad bridge and investigate how homogeneous and heterogeneous ballast assumptions affect the loading environment within the bridge approach. The main findings are summarized below:

- Ballast settlement in the approach attempts to create a ballast profile that reduces increased tie loads and evenly distributes the wheel load amongst underlying ties. This results in a condition in which the bridge approach is in an "equilibrium state".
- Assuming homogeneous ballast properties and settlement rates, minimal differential settlement is observed between the approach and open track.
- A sensitivity analysis that randomly varied the ballast settlement by +/- 0.5 mm at 16 and 28 MGT showed increased tie loads exceeding 80% within the bridge approach. This suggests slight variations in ballast condition can lead to increased tie loads within the transition zone.

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